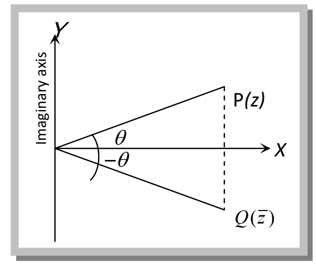
Conjugate of a Complex Number.

(1) **Conjugate complex number:** If there exists a complex number z = $a+ib$, $(a,b)\in$ R, then its conjugate is defined as $\bar{z} = a-ib$.



Hence, we have $\mathrm{Re}(z)=\frac{z+\bar{z}}{2}$ and $\mathrm{Im}(z)=\frac{z-\bar{z}}{2i}$. Geometrically, the conjugate of z is the reflection or point image of z in the real axis.

(2) **Properties of conjugate:** If z, z_1 and z_2 are existing complex numbers, then we have the following results:

(i) $\overline{(\bar{z})}=z$

(ii) $\overline{z_1+z_2}=\bar{z}_1+\bar{z}_2$

(iii) $\overline{z_1-z_2}=\bar{z}_1-\bar{z}_2$

(iv) $\overline{z_1 z_2}=\bar{z}_1\,\bar{z}_2$, In general $\overline{z_1.z_2.z_3.....z_n}=\bar{z}_1.\bar{z}_2.\bar{z}_3.....\bar{z}_n$

(v) $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2},\ z_2\neq 0$

(vi) $(\bar{z})^n=\overline{(z^n)}$

(vii) $z+\bar{z}=2\,\mathrm{Re}(z)=2\,\mathrm{Re}(\bar{z})=$ purely real

(viii) $z-\bar{z}=2i\,\mathrm{Im}(z)=$ purely imaginary

(ix) $z\bar{z}=$ purely real

(x) $z_1\bar{z}_2+\bar{z}_1 z_2=2\,\mathrm{Re}(z_1\bar{z}_2)=2\,\mathrm{Re}(\bar{z}_1 z_2)$

(xi) $z-\bar{z}=0$ i.e., $z=\bar{z}\Leftrightarrow z$ is purely real i.e., $\mathrm{Im}(z)=0$

(xii) $z + \bar{z} = 0$ i.e., $z = -\bar{z} \Leftrightarrow$ either $z = 0$ or z is purely imaginary i.e., $\text{Re}(z) = 0$

(xiii) $z_1 = z_2 \Leftrightarrow \bar{z}_1 = \bar{z}_2$

(xiv) $z = 0 \Leftrightarrow \bar{z} = 0$

(xv) $z\bar{z} = 0 \Leftrightarrow z = 0$

(xvi) If $w = f(z)$ then $\overline{w} = f(\bar{z})$

(xvii) $\overline{re^{i\theta}} = re^{-i\theta}$

Important Tips

- ☞ Complex conjugate is obtained by just changing the sign of i.
- ☞ Conjugate of $i = -i$
- ☞ Conjugate of $iz = -i\bar{z}$
- ☞ $(z_1 + z_2)$ and $(z_1 . z_2)$ real $\Leftrightarrow z_1 = \bar{z}_2$ or $z_2 = \bar{z}_1$
- ☞ $\overline{z_1 \bar{z}_2} = \bar{z}_1 z_2$

(3) **Reciprocal of a complex number :** For an existing non-zero complex number $z = a + ib$, the reciprocal is given by $z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$ i.e., $z^{-1} = \frac{1}{a+ib} \Rightarrow \frac{a-ib}{a^2+b^2} = \frac{\text{Re}(z)}{|z|^2} + \frac{i[-\text{Im}(z)]}{|z|^2} = \frac{\bar{z}}{|z|^2}$.