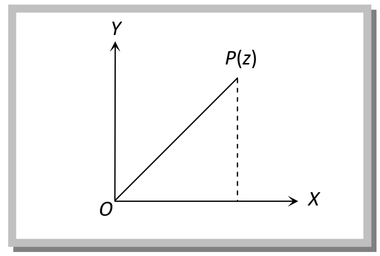
Modulus of a Complex Number.

Modulus of a complex number $z = a + ib$ is defined by a positive real number given by $|z| = \sqrt{a^2 + b^2}$, where a, b real numbers. Geometrically |z| represents the distance of point P (represented by z) from the origin,

i.e. |z| = OP.

If |z| = 0, then z is known as zero modular complex number and is used to represent the origin of reference plane.

If |z| = 1 the corresponding complex number is known as **unimodular complex number.** Clearly z lies on a circle of unit radius having centre (0, 0).



Note: In the set C of all complex numbers, the order relation is not defined. As such $z_1 > z_2 >$ or $z_1 < z_2$ has no meaning. But $|z_1| > |z_2|$ or $|z_1| < |z_2|$ has got its meaning since $|z_1|$ and $|z_2|$ are real numbers.

Properties of modulus

(i) $|z| \geq 0 \Rightarrow |z| = 0 \; iff \; z = 0$ and $|z| > 0 \; iff \; z \neq 0$.

(ii) $-|z| \leq \text{Re}(z) \leq |z|$ and $-|z| \leq \text{Im}(z) \leq |z|$

(iii) $|z| = |\bar{z}| = |-z| = |-\bar{z}| = |zi|$

(iv) $z\bar{z} = |z|^2 = |\bar{z}|^2$

(v) $|z_1 z_2| = |z_1||z_2|$. In general $|z_1 z_2 z_3 \ldots\ldots z_n| = |z_1||z_2||z_3| \ldots |z_n|$

(vi) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $(z_2 \neq 0)$

(vii) $|z^n| = |z|^n, n \in N$

(viii) $|z_1 \pm z_2|^2 = (z_1 \pm z_2)(\bar{z}_1 \pm \bar{z}_2) = |z_1|^2 + |z_2|^2 \pm (z_1\bar{z}_2 + \bar{z}_1 z_2)$ or $|z_1|^2 + |z_2|^2 \pm 2\,\text{Re}(z_1\bar{z}_2)$

(ix) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 \Leftrightarrow \frac{z_1}{z_2}$ is purely imaginary or $\text{Re}\left(\frac{z_1}{z_2}\right) = 0$

(x) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left\{|z_1|^2 + |z_2|^2\right\}$ (Law of parallelogram)

(xi) $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)\left(|z_1|^2 + |z_2|^2\right)$, where $a, b \in R$.

Important Tips

☞Modulus of every complex number is a non-negative real number.

☞$|z| = 0 \text{ iff } z = 0$ i.e., $\text{Re}(z) = \text{Im}(z) = 0$

☞$|z| \geq |\text{Re}(z)| \geq \text{Re}(z)$ and $|z| \geq |\text{Im}(z)| \geq \text{Im}(z)$

☞$|z| = 1 \Leftrightarrow \bar{z} = \frac{1}{z}$

☞$\left|\frac{z}{\bar{z}}\right| = 1$

☞$\frac{z}{|\bar{z}|}$ is always a unimodular complex number if $z \neq 0$

☞$\frac{z}{\bar{z}}$ is always a unimodular complex number if $z \neq 0$

☞$|\text{Re}(z)| + |\text{Im}(z)| \leq \sqrt{2}|z|$

☞$||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$

Thus $|z_1| + |z_2|$ is the greatest possible value of $|z_1 + z_2|$ and $||z_1| - |z_2||$ is the least possible value of $|z_1 + z_2|$

☞If $\left|z + \frac{1}{z}\right| = a$, the greatest and least values of $|z|$ are respectively $\frac{a + \sqrt{a^2 + 4}}{2}$ and $\frac{-a + \sqrt{a^2 + 4}}{2}$

☞$|z_1 + \sqrt{z_1^2 - z_2^2}| + |z_1 - \sqrt{z_1^2 - z_2^2}| = |z_1 + z_2| + |z_1 - z_2|$