Square Root of a Complex Number.

Let $a + ib$ be a complex number such that $\sqrt{a+ib} = x + iy,$ where x and y are real numbers.

Then

$\sqrt{a+ib} = x + iy \Rightarrow a + ib = (x+iy)^2 \Rightarrow a + ib = (x^2 - y^2) + 2ixy$

$\Rightarrow x^2 - y^2 = a$ .....(i)

and $2xy = b$ .....(ii) [On equating real and imaginary parts]

Solving, $x = \pm\sqrt{\left(\frac{\sqrt{a^2+b^2}+a}{2}\right)}$ and $y = \pm\sqrt{\left(\frac{\sqrt{a^2+b^2}-a}{2}\right)}$

$$\therefore \sqrt{a+ib} = \pm\left[\sqrt{\left(\frac{\sqrt{a^2+b^2}+a}{2}\right)} - i\sqrt{\left(\frac{\sqrt{a^2+b^2}-a}{2}\right)}\right]$$

Therefore $\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + i\sqrt{\frac{|z|-a}{2}}\right]$ for b>0 $= \pm\left[\sqrt{\frac{|z|+a}{2}} - i\sqrt{\frac{|z|-a}{2}}\right]$ for b<0.

Note: To find the square root of $a - ib,$ replace i by – i in the above results.

The square root of iis $\pm\left(\frac{1+i}{\sqrt{2}}\right)$, [Here b = 1]

The square root of – iis $\pm\left(\frac{1-i}{\sqrt{2}}\right)$, [Here b = –1]

Alternative method for finding the square root

(i) If the imaginary part is not even then multiply and divide the given complex number by 2. e.g. z = 8 – 15i here imaginary part is not even so write z = $\frac{1}{2}$ (16 – 30i) and let $a + ib$ = 16 – 30 i .

(ii) Now divide the numerical value of imaginary part of $a+ib$ by 2 and let quotient be P and find all possible two factors of the number P thus obtained and take that pair in which difference of squares of the numbers is equal to the real part of $a+ib$ e.g., here numerical value of Im(16 – 30i) is 30. Now 30 = 2×15 . All possible way to express 15 as a product of two are 1×15 , 3×5 etc. here $5^2 - 3^2$ = 16 = Re (16– 30i) so we will take 5, 3.

(iii) Take i with the smaller or the greater factor according as the real part of a + ib is positive or negative and if real part is zero then take equal factors of P and associate i with any one of them e.g., Re(16 – 30i) > 0, we will take i with 3. Now complete the square and write down the square root of z.

e.g., $z = \frac{1}{2}\left[16 - 30i\right] = \frac{1}{2}\left[5^2 + (3i)^2 - 2 \times 5 \times 3i\right] = \frac{1}{2}\left[5 - 3i\right]^2 \Rightarrow \sqrt{z} = \pm\frac{1}{\sqrt{2}}(5 - 3i)$