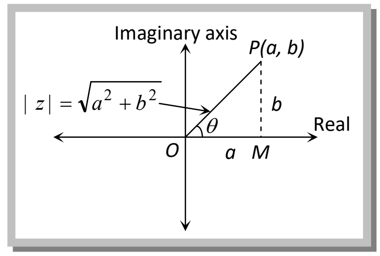
Representation of Complex Number.

A complex number can be represented in the following from:

(1) **Geometrical representation (Cartesian representation):** The complex number $z = a + ib = (a, b)$ is represented by a point P whose coordinates are referred to rectangular axes XOX' and YOY' which are called real and imaginary axis respectively. Thus a complex number z is represented by a point in a plane, and corresponding to every point in this plane there exists a complex number such a plane is called argand plane or argand diagram or complex plane or gaussian plane.



Note: Distance of any complex number from the origin is called the modules of complex number and is denoted by |z|, i.e., $|z| = \sqrt{a^2 + b^2}$

Angle of any complex number with positive direction of x– axis is called amplitude or argument of z. i.e., $amp\,(z) = arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$

(2) **Trigonometrical (Polar) representation:**In Δ OPM, let $OP = r$, then $a = r\cos\theta$ and $b = r\sin\theta$. Hence z can be expressed as $z = r(\cos\theta + i\sin\theta)$

Where r = |z| and θ = principal value of argument of z.

For general values of the argument $z = r[\cos(2n\pi + \theta) + i\sin(2n\pi + \theta)]$

Note: Sometimes $(\cos\theta + i\sin\theta)$ is written in short as $cis\theta$.

(3) **Vector representation**: If P is the point (a, b) on the argand plane corresponding to the complex number $z = a + ib$.

Then $\overrightarrow{OP} = a\hat{i} + b\hat{j}$, $\therefore |\overrightarrow{OP}| = \sqrt{a^2 + b^2} = |z|$ and arg z = direction of the vector $\overrightarrow{OP} = \tan^{-1}\left(\frac{b}{a}\right)$

Therefore, complex number z can also be represented by $\overrightarrow{OP}$.

(4) **Eulerian representation (Exponential form):** Since we have $e^{i\theta} = \cos\theta + i\sin\theta$ and thus z can be expressed as $z = re^{i\theta}$, where $|z| = r$ and $\theta =$ arg (z)

Note: $e^{-i\theta} = (\cos\theta - i\sin\theta)$

$e^{i\theta} + e^{-i\theta} = 2\cos\theta, e^{i\theta} - e^{-i\theta} = 2i\sin\theta$