Quadratic Expression.

An expression of the form $ax^2 + bx + c$, where a, b, c ∈ R and a ≠ 0 is called a quadratic expression in x. So in general, quadratic expression is represented by $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$.

(1) **Graph of a quadratic expression:** We have $y = ax^2 + bx + c = f(x)$

$$y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right] \Rightarrow y + \frac{D}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

Now, let $y + \frac{D}{4a} = Y$ and $X = x + \frac{b}{2a}$

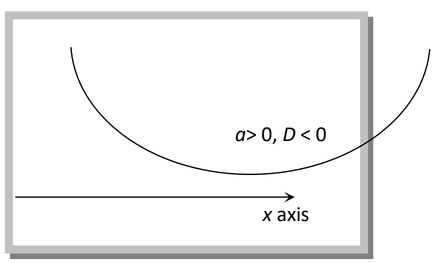
$$Y = a.X^2 \Rightarrow X^2 = \frac{1}{a}Y$$

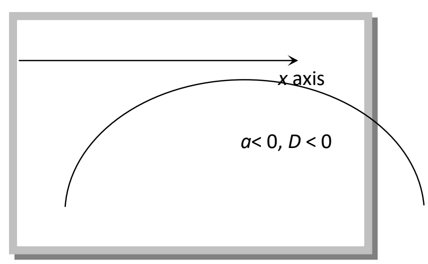
(i) The graph of the curve $y = f(x)$ is parabolic.

(ii) The axis of parabola is $X = 0$ or $x + \frac{b}{2a} = 0$ i.e. (parallel to y-axis).

(iii) (a) If a > 0, then the parabola opens upward.

(b) If a < 0, then the parabola opens downward.



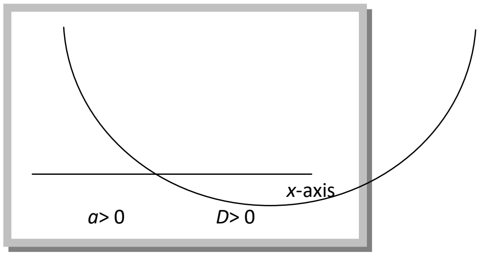


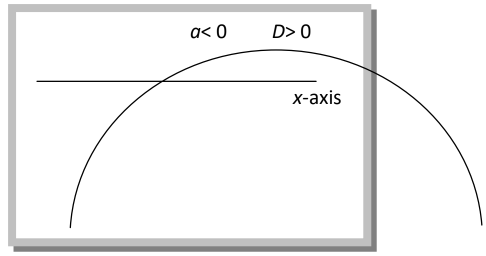
(iv) **Intersection with axis**

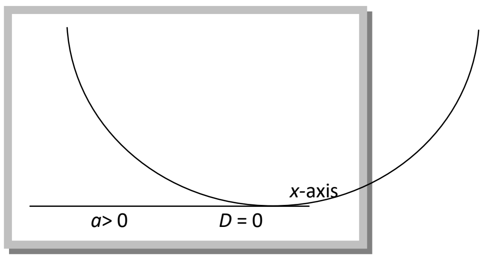
(a) **X-axis:** For x axis, $y = 0 \Rightarrow ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a}$

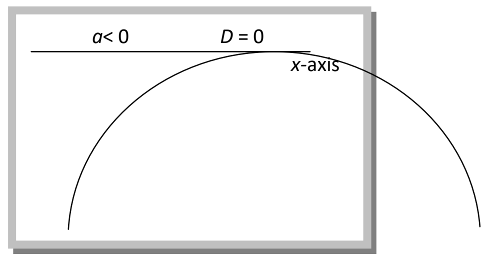
For D > 0, parabola cuts x-axis in two real and distinct point's i.e. $x = \frac{-b \pm \sqrt{D}}{2a}$.

For D = 0, parabola touches x-axis in one point, $x = -b/2a$.

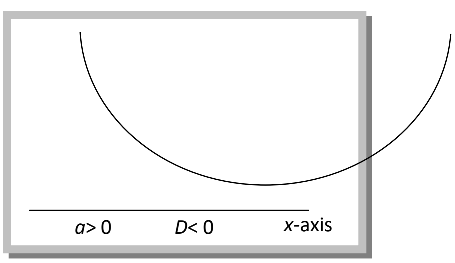


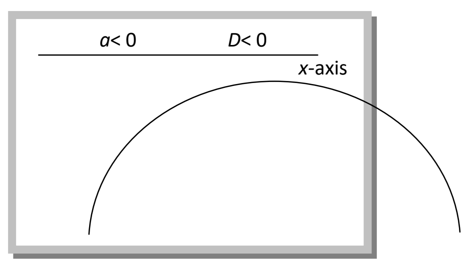






For D < 0, parabola does not cut x-axis(i.e. imaginary value of x).





(b) **y-axis:**For y axis $x = 0$, $y = c$

(2) **Maximum and minimum values of quadratic expression:**Maximum and minimum value of quadratic expression can be found out by two methods:

(i) **Discriminant method:**In a quadratic expression $ax^2 + bx + c$.

(a) If a > 0, quadratic expression has least value at $x = -b/2a$. This least value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$.

(b) If a < 0, quadratic expression has greatest value at $x = -b/2a$. This greatest value is given by $\frac{4ac - b^2}{4a} = -\frac{D}{4a}$.

(ii) **Graphical method:** Vertex of the parabola $Y = aX^2$ is $X = 0$, $Y = 0$

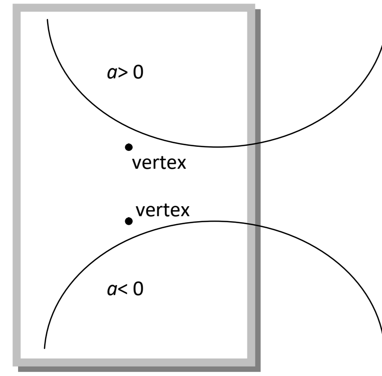
i.e., $x + \frac{b}{2a} = 0$, $y + \frac{D}{4a} = 0 \Rightarrow x = -b/2a$, $y = -D/4a$

Hence, vertex of $y = ax^2 + bx + c$ is $(-b/2a, -D/4a)$

(a) For a > 0, f(x) has least value at $x = -\frac{b}{2a}$. This least value is given by

$f\left(-\frac{b}{2a}\right) = -\frac{D}{4a}$.

(b) For a < 0, f(x) has greatest value at $x = -b/2a$. This greatest value is given by $f\left(-\frac{b}{2a}\right) = -\frac{D}{4a}$.



(3) **Sign of quadratic expression:** Let $f(x) = ax^2 + bx + c$ or $y = ax^2 + bx + c$

Where a, b, c $\in$ R and a $\neq$ 0, for some values of x, f(x) may be positive, negative or zero.

This gives the following cases:

(i) a> 0 and D < 0, so $f(x) > 0$ for all $x \in R$ i.e., $f(x)$ is positive for all real values of x.

(ii) a< 0 and D < 0, so $f(x) < 0$ for all x $\in$ R i.e., f(x) is negative for all real values of x.

(iii) a> 0 and D = 0 so, $f(x) \geq 0$ for all x $\in$ R i.e., f(x) is positive for all real values of x except at vertex, where $f(x) = 0$.

(iv) a< 0 and D = 0 so, $f(x) \leq 0$ for all x $\in$ R i.e. f(x) is negative for all real values of x except at vertex, where $f(x) = 0$.

(v) a> 0 and D > 0

Let $f(x) = 0$ have two real roots α and β $(\alpha < \beta)$, then $f(x) > 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) < 0$ for all $x \in (\alpha, \beta)$.

(vi) $a < 0$ and $D > 0$

Let $f(x) = 0$ have two real roots α and β $(\alpha < \beta)$,

Then $f(x) < 0$ for all $x \in (-\infty, \alpha) \cup (\beta, \infty)$ and $f(x) > 0$ for all $x \in (\alpha, \beta)$