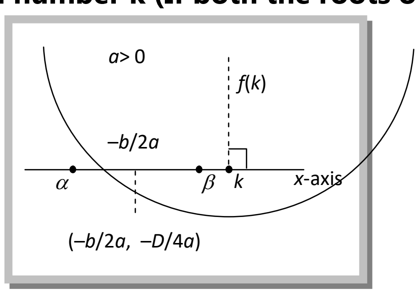
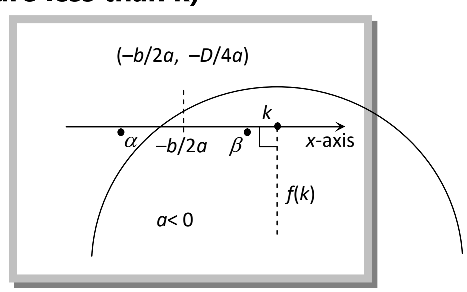
Position of Roots of a Quadratic Equation.

Let $f(x) = ax^2 + bx + c$, where a, b, c $\in$ R be a quadratic expression and k, k_1, k_2 be real numbers such that $k_1 < k_2$. Let α, β be the roots of the equation $f(x) = 0$ i.e. $ax^2 + bx + c = 0$. Then $\alpha = \dfrac{-b+\sqrt{D}}{2a}$, $\beta = \dfrac{-b-\sqrt{D}}{2a}$ where D is the discriminant of the equation.

(1) **Condition for a number k (If both the roots of f(x) = 0 are less than k)**

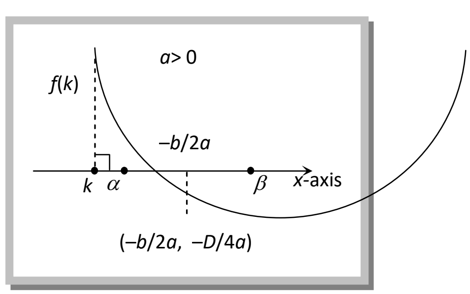


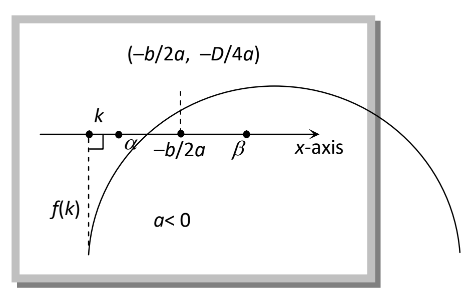


(i) $D \geq 0$ (roots may be equal) (ii) $a\,f(k) > 0$ (iii) $k > -b/2a$

, where $\alpha \leq \beta$

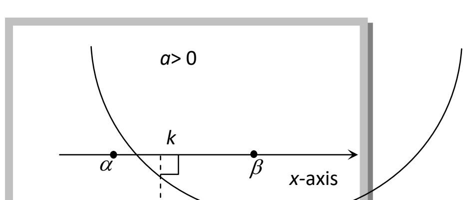
(2) **Condition for a number k (If both the roots of f(x) = 0 are greater than k)**

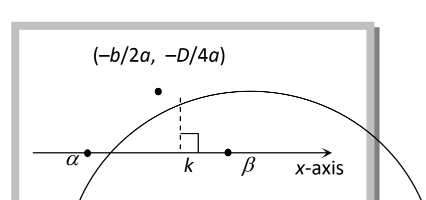




(i) $D \geq 0$ (roots may be equal) (ii) $a\,f(k) > 0$ (iii) $k < -b/2a$

, where $\alpha \leq \beta$



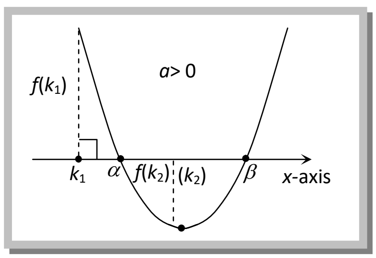


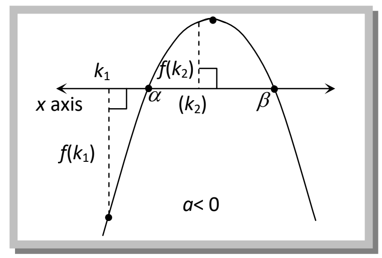
(3) **Condition for a number k (If k lies between the roots of f(x) = 0)**

(i) $D > 0$

(ii) $a\,f(k) < 0$, where $\alpha < \beta$

(4) **Condition for numbers k_1 and k_2 (If exactly one root of f(x) = 0 lies in the interval (k_1, k_2))**

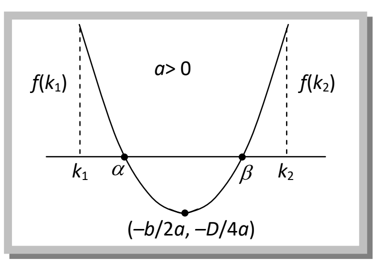


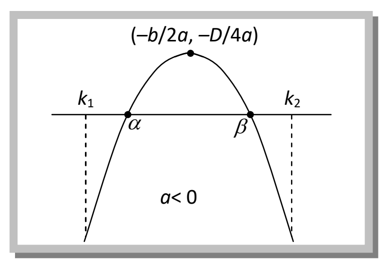


(i) $D > 0$

(ii) $f(k_1)f(k_2) < 0$, where $\alpha < \beta$.

(5) **Condition for numbers k_1 and k_2 (If both roots of f(x) = 0 are confined between k_1 and k_2)**





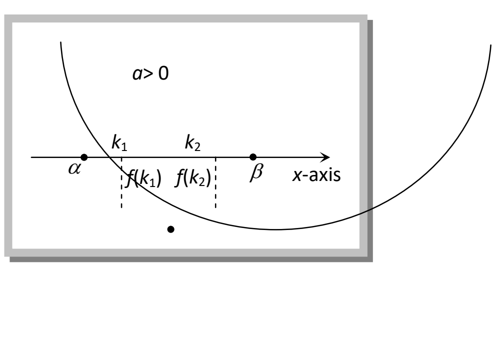
(i) $D \geq 0$ (roots may be equal)

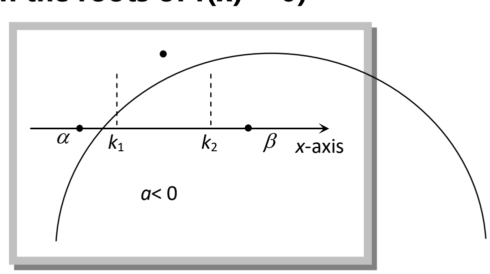
(ii) $a\,f(k_1) > 0$

(iii) $a\,f(k_2) > 0$

(iv) $k_1 < -b/2a < k_2$, where $\alpha \leq \beta$ and $k_1 < k_2$

(6) **Condition for numbers k_1 and k_2 (If k_1 and k_2 lie between the roots of f(x) = 0)**





(i) $D > 0$ (ii) $a\,f(k_1) < 0$ (iii) $a\,f(k_2) < 0$,

where $\alpha < \beta$