Rational Algebraic Inequations.

(1) **Values of rational expression P(x)/Q(x) for real values of x, where P(x) and Q(x) are quadratic expressions:** To find the values attained by rational expression of the form $\frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2}$ for real values of x, the following algorithm will explain the procedure :

Algorithm

Step I: Equate the given rational expression to y.

Step II: Obtain a quadratic equation in x by simplifying the expression in step I.

Step III: Obtain the discriminant of the quadratic equation in Step II.

Step IV: Put Discriminant ≥ 0 and solve the inequations for y. The values of y so obtained determines the set of values attained by the given rational expression.

(2) **Solution of rational algebraic inequations:** If P(x) and Q(x) are polynomial in x, then the inequations $\frac{P(x)}{Q(x)} > 0, \frac{P(x)}{Q(x)} < 0, \frac{P(x)}{Q(x)} \geq 0$ and $\frac{P(x)}{Q(x)} \leq 0$ are known as rational algebraic inequations.

To solve these inequations we use the sign method as explained in the following algorithm.

Algorithm

Step I: Obtain P(x) and Q(x).

Step II: Factorize P(x) and Q(x) into linear factors.

Step III: Make the coefficient of x positive in all factors.

Step IV: Obtain critical points by equating all factors to zero.

Step V: Plot the critical points on the number line. If there are n critical points, they divide the number line into (n + 1) regions.

Step VI: In the right most region the expression $\frac{P(x)}{Q(x)}$ bears positive sign and in other regions the expression bears positive and negative signs depending on the exponents of the factors.