Algebraic Interpretation of Rolle's Theorem.

Let f(x) be a polynomial having α and β as its roots, such that $\alpha < \beta$. Then, $f(\alpha) = f(\beta) = 0$. Also a polynomial function is everywhere continuous and differentiable. Thus f(x) satisfies all the three conditions of Rolle 's Theorem. Consequently there exists $\gamma \in (\alpha, \beta)$ such that $f'(\gamma) = 0$ i.e. $f'(x) = 0$ at $x = \gamma$. In other words $x = \gamma$ is a root of $f'(x) = 0$. Thus algebraically Rolle 's Theorem can be interpreted as follows.

Between any two roots of polynomial f(x), there is always a root of its derivative $f'(x)$.

Lagrange's theorem: Let f(x) be a function defined on [a b] such that

(i) f(x) is continuous on [a b] and

(ii) f(x) is differentiable on (a, b), then c $\in$ (a, b), such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Lagrange's identity: If $a_1, a_2, a_3, b_1, b_2, b_3 \in R$ then :

$$(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1 b_1 + a_2 b_2 + a_3 b_3)^2 = (a_1 b_2 - a_2 b_1)^2 + (a_2 b_3 - a_3 b_2)^2 + (a_3 b_1 - a_1 b_3)^2$$