Equation and Inequation containing Absolute Value

(1) **Equations containing absolute values**

By definition, $|x| = \begin{cases} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{cases}$

Important forms containing absolute value:

Form I: The equation of the form $|f(x)+g(x)| = |f(x)| + |g(x)|$ is equivalent of the system $f(x).g(x) \geq 0$.

Form II: The equation of the form $|f_1(x)| + |f_2(x)| + |f_3(x)| + \ldots\ldots |f_n(x)| = g(x)$ .....(i)

Where $f_1(x), f_2(x), f_3(x)\ldots\ldots f_n(x), g(x)$ are functions of x and g(x) may be a constant.

Equations of this form can be solved by the method of interval. We first find all critical points of $f_1(x), f_2(x)\ldots\ldots f_n(x)$. If coefficient of x is +ve, then graph starts with +ve sign and if it is negative, then graph starts with negative sign. Then using the definition of the absolute value, we pass form equation (i) to a collection of system which do not contain the absolute value symbols.

(2) **Inequations containing absolute value**

By definition, |x| < a $\Rightarrow -a < x < a$ (a > 0), $|x| \leq a \Rightarrow -a \leq x \leq a$,

|x| > a $\Rightarrow x < -a$ or $x > a$ and $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$