Nature of Roots.

In quadratic equation $ax^2 + bx + c = 0$, the term $b^2 - 4ac$ is called discriminant of the equation, which plays an important role in finding the nature of the roots. It is denoted by Δ or D.

(1) **If a, b, c ∈ R and a ≠ 0, then:**(i) If D < 0, then equation $ax^2 + bx + c = 0$ has non-real complex roots.

(ii) If D > 0, then equation $ax^2 + bx + c = 0$ has real and distinct roots, namely $\alpha = \frac{-b + \sqrt{D}}{2a}$, $\beta = \frac{-b - \sqrt{D}}{2a}$

and then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$ .....(i)

(iii) If D = 0, then equation $ax^2 + bx + c = 0$ has real and equal roots $\alpha = \beta = \frac{-b}{2a}$

and then $ax^2 + bx + c = a(x - \alpha)^2$ .....(ii)

To represent the quadratic expression $ax^2 + bx + c$ in form (i) and (ii), transform it into linear factors.

(iv) If D ≥ 0, then equation $ax^2 + bx + c = 0$ has real roots.

(2) **If a, b, c ∈ Q, a ≠ 0, then:**(i) If D > 0 and D is a perfect square ⇒ roots are unequal and rational.

(ii) If D > 0 and D is not a perfect square ⇒ roots are irrational and unequal.

(3) **Conjugate roots:**The irrational and complex roots of a quadratic equation always occur in pairs. Therefore

(i) If one root be $\alpha + i\beta$ then other root will be $\alpha - i\beta$.

(ii) If one root be $\alpha + \sqrt{\beta}$ then other root will be $\alpha - \sqrt{\beta}$.

(4) **If D_1 and D_2 be the discriminants of two quadratic equations,then**

(i) If $D_1 + D_2 \geq 0$, then

(a) At least one of D_1 and $D_2 \geq 0$.

(b) If $D_1 < 0$ then $D_2 > 0$

(ii) If $D_1 + D_2 < 0$, then

(a) At least one of D_1 and $D_2 < 0$.

(b) If $D_1 > 0$ then $D_2 < 0$.