Roots under Particular Conditions.

For the quadratic equation $ax^2 + bx + c = 0$.

(1) If $b = 0 \Rightarrow$ roots are of equal magnitude but of opposite sign.

(2) If $c = 0 \Rightarrow$ one root is zero, other is $-b/a$.

(3) If $b = c = 0 \Rightarrow$ both roots are zero.

(4) If $a = c \Rightarrow$ roots are reciprocal to each other.

(5) If $\left.\begin{matrix} a > 0 & c < 0 \\ a < 0 & c > 0 \end{matrix}\right\} \Rightarrow$ roots are of opposite signs.

(6) If $\left.\begin{matrix} a > 0 & b > 0 & c > 0 \\ a < 0 & b < 0 & c < 0 \end{matrix}\right\} \Rightarrow$ both roots are negative, provided $D \geq 0$.

(7) If $\left.\begin{matrix} a > 0 & b < 0 & c > 0 \\ a < 0 & b > 0 & c < 0 \end{matrix}\right\} \Rightarrow$ both roots are positive, provided $D \geq 0$.

(8) If sign of a = sign of b $\neq$ sign of c $\Rightarrow$ greater root in magnitude, is negative.

(9) If sign of b = sign of c $\neq$ sign of a $\Rightarrow$ greater root in magnitude, is positive.

(10) If $a + b + c = 0 \Rightarrow$ one root is 1 and second root is c/a.

(11) If $a = b = c = 0$, then equation will become an identity and will be satisfied by every value of x.

(12) If $a = 1$ and b, c $\in$ I and the root of equation $ax^2 + bx + c = 0$ are rational numbers, then these roots must be integers.

Important Tips

- ☞ If an equation has only one change of sign, it has one +ve root and no more.
- ☞ If all the terms of an equation are +ve and the equation involves no odd power of x, then all its roots are complex.