# Relations between Roots and Coefficients.

(1) **Relation between roots and coefficients of quadratic equation:** If α and β are the roots of quadratic equation $ax^2 + bx + c = 0$ , (a ≠ 0) then

Sum of roots $= S = \alpha + \beta = \frac{-b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of roots $= P = \alpha.\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

If roots of quadratic equation $ax^2 + bx + c = 0$ (a ≠ 0) are α and β then

(i) $(\alpha - \beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \pm\frac{\sqrt{b^2 - 4ac}}{a} = \frac{\pm\sqrt{D}}{a}$

(ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$

(iii) $\alpha^2 - \beta^2 = (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = -\frac{b\sqrt{b^2 - 4ac}}{a^2} = \frac{\pm b\sqrt{D}}{a^2}$

(iv) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = -\frac{b(b^2 - 3ac)}{a^3}$

(v) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta) = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\,\{(\alpha+\beta)^2 - \alpha\beta\} = \frac{\pm(b^2 - ac)\sqrt{b^2 - 4ac}}{a^3}$

(vi) $\alpha^4 + \beta^4 = \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\frac{b^2 - 2ac}{a^2}\right)^2 - 2\frac{c^2}{a^2}$

(vii) $\alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = \frac{\pm b(b^2 - 2ac)\sqrt{b^2 - 4ac}}{a^4}$

(viii) $\alpha^2 + \alpha\beta + \beta^2 = (\alpha+\beta)^2 - \alpha\beta = \frac{b^2 - ac}{a^2}$

(ix) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{b^2 - 2ac}{ac}$

(x) $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha+\beta) = -\frac{bc}{a^2}$

(xi) $\left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 = \frac{a^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2} = \frac{b^2 D + 2a^2c^2}{a^2c^2}$

(2) **Formation of an equation with given roots:** A quadratic equation whose roots are $\alpha$ and $\beta$ is given by $(x-\alpha)(x-\beta)=0$

$\therefore x^2-(\alpha+\beta)x+\alpha\beta=0$ i.e. $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$\therefore x^2-Sx+P=0$

(3) **Equation in terms of the roots of another equation:** If $\alpha$, $\beta$ are roots of the equation $ax^2+bx+c=0$, then the equation whose roots are

(i) $-\alpha, -\beta \Rightarrow ax^2-bx+c=0$ (Replace x by – x)

(ii) $1/\alpha, 1/\beta \Rightarrow cx^2+bx+a=0$ (Replace x by 1/x)

(iii) $\alpha^n, \beta^n$; $n \in \mathbb{N} \Rightarrow a(x^{1/n})^2+b(x^{1/n})+c=0$ (Replace x by $x^{1/n}$)

(iv) $k\alpha, k\beta \Rightarrow ax^2+kbx+k^2c=0$ (Replace x by x/k)

(v) $k+\alpha, k+\beta \Rightarrow a(x-k)^2+b(x-k)+c=0$ (Replace x by (x – k))

(vi) $\frac{\alpha}{k}, \frac{\beta}{k} \Rightarrow k^2ax^2+kbx+c=0$ (Replace x by kx)

(vii) $\alpha^{1/n}, \beta^{1/n}$; $n \in \mathbb{N} \Rightarrow a(x^n)^2+b(x^n)+c=0$ (Replace x by $x^n$)

(4) **Symmetric expressions:** The symmetric expressions of the roots $\alpha$, $\beta$ of an equation are those expressions in $\alpha$ and $\beta$, which do not change by interchanging $\alpha$ and $\beta$. To find the value of such an expression, we generally express that in terms of $\alpha+\beta$ and $\alpha\beta$.

Some examples of symmetric expressions are:

(i) $\alpha^2+\beta^2$

(ii) $\alpha^2+\alpha\beta+\beta^2$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$

(iv) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

(v) $\alpha^2\beta+\beta^2\alpha$

(vi) $\left(\frac{\alpha}{\beta}\right)^2+\left(\frac{\beta}{\alpha}\right)^2$

(vii) $\alpha^3 + \beta^3$

(viii) $\alpha^4 + \beta^4$