Condition for Common Roots.

(1) **Only one root is common:** Let α be the common root of quadratic equations $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$.

$\therefore a_1\alpha^2+b_1\alpha+c_1=0$, $a_2\alpha^2+b_2\alpha+c_2=0$

By Crammer's rule : $\dfrac{\alpha^2}{\begin{vmatrix}-c_1 & b_1\\ -c_2 & b_2\end{vmatrix}}=\dfrac{\alpha}{\begin{vmatrix}a_1 & -c_1\\ a_2 & -c_2\end{vmatrix}}=\dfrac{1}{\begin{vmatrix}a_1 & b_1\\ a_2 & b_2\end{vmatrix}}$ or $\dfrac{\alpha^2}{b_1c_2-b_2c_1}=\dfrac{\alpha}{a_2c_1-a_1c_2}=\dfrac{1}{a_1b_2-a_2b_1}$

$\therefore \alpha=\dfrac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}=\dfrac{b_1c_2-b_2c_1}{a_2c_1-a_1c_2}$, $\alpha\neq 0$

$\therefore$ The condition for only one root common is $(c_1a_2-c_2a_1)^2=(b_1c_2-b_2c_1)(a_1b_2-a_2b_1)$

(2) **Both roots are common:** Then required condition is $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$.

Important Tips

- ☞ To find the common root of two equations, make the coefficient of second degree term in the two equations equal and subtract. The value of x obtained is the required common root.
- ☞ Two different quadratic equations with rational coefficient can not have single common root which is complex or irrational as imaginary and surd roots always occur in pair.