

An Important Theorem.

If $(\sqrt{A} + B)^n = I + f$ where I and n are positive integers, n being odd and $0 \leq f < 1$ then $(I + f) \cdot f = K^n$ where $A - B^2 = K > 0$ and $\sqrt{A} - B < 1$.

Note: If n is even integer then $(\sqrt{A} + B)^n + (\sqrt{A} - B)^n = I + f + f'$

Hence L.H.S. and I are integers.

$\therefore f + f'$ is also integer; $\Rightarrow f + f' = 1$; $\therefore f' = (1 - f)$

Hence $(I + f)(1 - f) = (I + f)f' = (\sqrt{A} + B)^n (\sqrt{A} - B)^n = (A - B^2)^n = K^n$.