Some Important Expansions.

(1) Replacing y by $-y$ in (i), we get,

$$(x-y)^n = {}^nC_0\, x^{n-0}.y^0 - {}^nC_1 x^{n-1}.y^1 + {}^nC_2 x^{n-2}.y^2 .... + (-1)^r\ {}^nC_r x^{n-r}.y^r + ... + (-1)^n\ {}^nC_n x^0.y^n$$

i.e., $(x-y)^n = \sum_{r=0}^{n} (-1)^r\ {}^nC_r x^{n-r}.y^r$ .....(ii)

The terms in the expansion of $(x-y)^n$ are alternatively positive and negative, the last term is positive or negative according as n is even or odd.

(2) Replacing x by 1 and y by x in equation (i) we get,

$(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + ... + {}^nC_r x^r + ... + {}^nC_n x^n$ *i.e.,* $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$

This is expansion of $(1+x)^n$ in ascending power of x.

(3) Replacing x by 1 and y by $-x$ in (i) we get,

$$(1-x)^n = {}^nC_0 x^0 - {}^nC_1 x^1 + {}^nC_2 x^2 - ...... + (-1)^r\ {}^nC_r x^r + ... + (-1)^n\ {}^nC_n x^n$$ *i.e.,*

$(1-x)^n = \sum_{r=0}^{n} (-1)^r\ {}^nC_r x^r$

(4) $(x+y)^n + (x-y)^n = 2[{}^nC_0 x^n y^0 + {}^nC_2 x^{n-2} y^2 + {}^nC_4 x^{n-4} y^4 + ...]$ and

$(x+y)^n - (x-y)^n = 2[{}^nC_1 x^{n-1} y^1 + {}^nC_3 x^{n-3} y^3 + {}^nC_5 x^{n-5} y^5 + ...]$

(5) The coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n$ is ${}^nC_r$.

(6) The coefficient of x^r in the expansion of $(1+x)^n$ is ${}^nC_r$.

Note: If n is odd, then $(x+y)^n+(x-y)^n$ and $(x+y)^n-(x-y)^n$, both have the same number of terms equal to $\left(\frac{n+1}{2}\right)$.

If n is even, then $(x+y)^n+(x-y)^n$ has $\left(\frac{n}{2}+1\right)$ terms and $(x+y)^n-(x-y)^n$ has $\frac{n}{2}$ terms.