1. Definitions.

Let A and B be two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that a is related to b by the relation R and write it as $a\,R\,b$. If $(a,b) \in R$, we write it as $a\,R\,b$.

Example: Let $A = \{1, 2, 5, 8, 9\}$, $B = \{1, 3\}$ we set a relation from A to B as: $a\,R\,b$ iff $a \le b$; $a \in A,\ b \in B$. Then $R = \{(1, 1)\}, (1, 3), (2, 3)\} \subset A \times B$

(1) **Total number of relations:** Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subset of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines relation from A to B, so total number of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation $A \times B$ are trivial relations from A to B.

(2) **Domain and range of a relation:** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Dom $(R) = \{a : (a, b) \in R\}$ and Range $(R) = \{b : (a, b) \in R\}$.

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

(3) **Relation on a set:** Let A be a non-void set. Then, a relation from A to itself *i.e.* a subset of $A \times A$ is called a relation on set A.