1. Equivalence Classes of an Equivalence Relation.

Let R be equivalence relation in $A(\neq \phi)$. Let $a \in A$. Then the equivalence class of a, denoted by [a] or $\{\overline{a}\}$ is defined as the set of all those points of A which are related to a under the relation R. Thus [a] = {x ∈A : x R a}.

It is easy to see that

(1) $b \in [a] \Rightarrow a \in [b]$ (2) $b \in [a] \Rightarrow [a] = [b]$ (3) Two equivalence classes are either disjoint or identical.

As an example we consider a very important equivalence relation $x \equiv y(\mathrm{mod}\ n)$ iff n divides $(x - y), n$ is a fixed positive integer. Consider $n = 5$. Then

[0] = {x : x ≡ 0 (mod 5)} = {5p : p ∈ Z} = $\{0, \pm 5, \pm 10, \pm 15, ...\}$

[1] = $\{x : x \equiv 1(\mathrm{mod}\ 5)\} = \{x : x - 1 = 5k, k \in Z\} = \{5k + 1 : k \in Z\} = \{1, 6, 11, ..., -4, -9, ...\}$.

One can easily see that there are only 5 distinct equivalence classes viz. [0], [1], [2], [3] and [4], when n = 5.