1. Axiomatic Definitions of the Set of Natural Numbers (Peano's Axioms).

The set N of natural numbers (N = {1, 2, 3, 4……}) is a set satisfying the following axioms (known as peano's axioms)

(1) N is not empty.

(2) There exist an injective (one-one) map $S : N \to N$ given by $S(n) = n^{+}$, where n^{+} is the immediate successor of n in N i.e., $n + 1 = n^{+}$.

(3) The successor mapping S is not surjective (onto).

(4) If $M \subseteq N$ such that,

(i) M contains an element which is not the successor of any element in N, and

(ii) $m \in M \Rightarrow m^{+} \in M$, then $M = N$

This is called the axiom of induction. We denote the unique element which is not the successor of any element is 1. Also, we get $1^{+} = 2, 2^{+} = 3$.

Note: Addition in N is defined as,

$n + 1 = n^{+}$

$n + m^{+} = (n + m)^{+}$

Multiplication in N is defined by,

$n.1 = n$

$n.m^{+} = n.m + n$