

## Exponential Function $a^x$ , where $a > 0$ .

$$\therefore a^x = e^{\log_e a^x} = e^{x \log_e a}$$

$$\therefore a^x = e^{\alpha x} \quad \dots \text{(i)}, \text{ where } \alpha = \log_e a. \text{ We have, } e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^r}{r!} + \dots \infty$$

$$\text{Replacing } x \text{ by } \alpha x \text{ in this series, } e^{\alpha x} = 1 + \frac{\alpha x}{1!} + \frac{\alpha^2 x^2}{2!} + \frac{\alpha^3 x^3}{3!} + \dots + \frac{\alpha^r x^r}{r!} + \dots \infty$$

$$\text{Hence from (i), } a^x = 1 + \frac{\log_e a}{1!} x + \frac{(\log_e a)^2}{2!} x^2 + \dots + \frac{(\log_e a)^r x^r}{r!} + \dots \infty$$