## Some Standard results.

(1) $\sum_{n=0}^{\infty}\frac{1}{n!}=\sum_{n=0}^{\infty}\frac{1}{(n-1)!}=\sum_{n=0}^{\infty}\frac{1}{(n-k)!}=e$

(2) $\sum_{n=1}^{\infty}\frac{1}{n!}=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\dots\dots\dots\infty=e-1$

(3) $\sum_{n=2}^{\infty}\frac{1}{n!}=\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots\dots\dots\infty=e-2$

(4) $\sum_{n=0}^{\infty}\frac{1}{(n+1)!}=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\dots\dots\dots\infty=e-1$

(5) $\sum_{n=0}^{\infty}\frac{1}{(n+2)!}=\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots\dots\dots\infty=e-2$

(6) $\sum_{n=1}^{\infty}\frac{1}{(n+1)!}=\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots\dots\dots\infty=e-2$

(7) $\sum_{n=0}^{\infty}\frac{1}{(2n)!}=1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\dots\dots\dots\infty=\frac{e+e^{-1}}{2}=\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}$

(8) $\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}=\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\dots\dots\dots\infty=\frac{e-e^{-1}}{2}=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}$

(9) $e^{x}=1+\frac{x}{1!}+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\dots\dots\dots+\frac{x^{n}}{n!}+\dots\dots\dots\infty$

$\therefore T_{n+1}=$ General term in the expansion of $e^{x}=\frac{x^{n}}{n!}$ and coefficient of $x^{n}$ in $e^{x}=\frac{1}{n!}$

(10) $e^{-x}=1-\frac{x}{1!}+\frac{x^{2}}{2!}-\frac{x^{3}}{3!}+\dots\dots\dots+(-1)^{n}\frac{x^{n}}{n!}+\dots\dots\dots\infty$

$\therefore T_{n+1} =$ General term in the expansion of $e^{-x} = (-1)^n \dfrac{x^n}{n!}$ and coefficient of $x^n$ in $e^{-x} = \dfrac{(-1)^n}{n!}$

(11) $e^{ax} = 1 + \dfrac{(ax)}{1!} + \dfrac{(ax)^2}{2!} + \dfrac{(ax)^3}{3!} + \dots\dots\dots + \dfrac{(ax)^n}{n!} + \dots\dots\dots \infty$

$\therefore T_{n+1} =$ General term in the expansion of $e^{ax} = \dfrac{(ax)^n}{n!}$ and coefficient of $x^n$ in $e^{ax} = \dfrac{a^n}{n!}$

(12) $\sum_{n=0}^{\infty} \dfrac{n}{n!} = e = \sum_{n=1}^{\infty} \dfrac{n}{n!}$

(13) $\sum_{n=0}^{\infty} \dfrac{n^2}{n!} = 2e = \sum_{n=1}^{\infty} \dfrac{n^2}{n!}$

(14) $\sum_{n=0}^{\infty} \dfrac{n^3}{n!} = 5e = \sum_{n=1}^{\infty} \dfrac{n^3}{n!}$

(15) $\sum_{n=0}^{\infty} \dfrac{n^4}{n!} = 15e = \sum_{n=1}^{\infty} \dfrac{n^4}{n!}$