Some Important Results from the Logarithmic Series.

(1) Replacing x by $-x$ in the logarithmic series, we get

$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\ldots\ldots \infty$ or $-\log_e(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\ldots\ldots \infty$

(2) (i) $\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2\left\{\frac{x^2}{2} + \frac{x^4}{4} + \ldots\ldots\ldots \infty\right\}, (-1 < x < 1)$

(ii) $\log_e(1+x) - \log_e(1-x) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\ldots \infty\right]$ or $\log_e\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\ldots \infty\right]$

(3) The series expansion of $\log_e(1+x)$ may fail to be valid if |x| is not less than 1. It can be proved that the logarithmic series is valid for x=1. Putting x=1 in the logarithmic series.

We get, $\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots\ldots\ldots \infty = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots\ldots\ldots \infty$

(4) When $x = -1$, the logarithmic series does not have a sum. This is in conformity with the fact that log (1-1) is not a finite quantity.