Some Special Determinants.

(1) **Symmetric determinant:** A determinant is called symmetric determinant if for its every element

$a_{ij} = a_{ji} \forall\ i, j$ e.g., $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$

(2) **Skew-symmetric determinant:** A determinant is called skew symmetric determinant if for its every element $a_{ij} = -a_{ji} \forall i, j$ e.g., $\begin{vmatrix} 0 & 3 & -1 \\ -3 & 0 & 5 \\ 1 & -5 & 0 \end{vmatrix}$

Note: Every diagonal element of a skew symmetric determinant is always zero.

The value of a skew symmetric determinant of even order is always a perfect square and that of odd order is always zero.

(ii) $\begin{vmatrix} 0 & a \\ -a & 0 \end{vmatrix} = 0 + a^2 = a^2$ (Perfect square)

(iii) $\begin{vmatrix} 0 & a-b & e-f \\ b-a & 0 & l-m \\ f-e & m-l & 0 \end{vmatrix} = 0$

(3) **Cyclic order:** If elements of the rows (or columns) are in cyclic order.

i.e.

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

(iii) $\begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

(iv) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

(v) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$

Note: These results direct applicable in lengthy questions (As behavior of standard results)