Expansion of Determinants.

Unlike a matrix, determinant is not just a table of numerical data but (quite differently) a short hand way of writing algebraic expression, whose value can be computed when the values of terms or elements are known.

(1) The 4 numbers a_1, b_1, a_2, b_2 arranged as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is a determinant of second order. These numbers are called elements of the determinant. The value of the determinant is defined as $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$.

The expanded form of determinant has 2! terms.

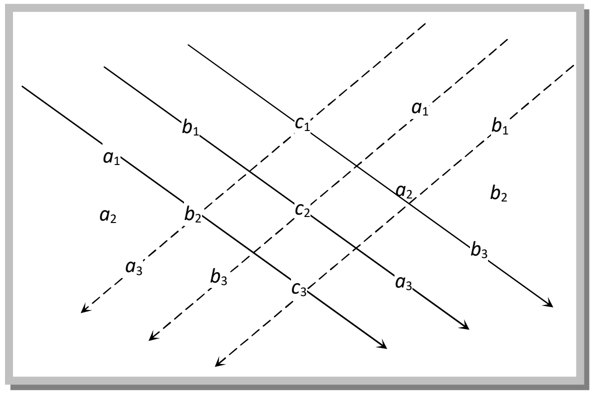
(2) The 9 numbers $a_r,\ b_r,\ c_r\ (r = 1,\ 2,\ 3)$ arranged as $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is a determinant of third order.

Take any row (or column); the value of the determinant is the sum of products of the elements of the row (or column) and the corresponding determinant obtained by omitting the row and the column of the element with a proper sign, given by the rule $(-1)^{i+j}$, where i and j are the number of rows and the number of columns respectively of the element of the row (or the column) chosen.

Thus $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

The diagonal through the left-hand top corner which contains the element $a_1,\ b_2,\ c_3$ is called the leading diagonal or principal diagonal and the terms are called the leading terms. The expanded form of determinant has 3! terms.

Short cut method or Sarrus diagram method: To find the value of third order determinant, following method is also useful



Taking product of R.H.S. diagonal elements positive and L.H.S. diagonal elements negative and adding them. We get the value of determinant as

$= a_1 b_2 c_3 + b_1 c_2 a_3 + c_1 a_2 b_3 - c_1 b_2 a_3 - a_1 c_2 b_3 - b_1 a_2 c_3$

Note: This method does not work for determinants of order greater than three.