Properties of Determinants.

P-1: The value of determinant remains unchanged, if the rows and the columns are interchanged.

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$. Then $D' = D$, D and D' are transpose of each other.

Note: Since the determinant remains unchanged when rows and columns are interchanged, it is obvious that any theorem which is true for 'rows' must also be true for 'columns'.

P-2: If any two rows (or columns) of a determinant be interchanged, the determinant is unaltered in numerical value but is changed in sign only.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$. Then $D' = -D$

P-3: If a determinant has two rows (or columns) identical, then its value is zero.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$. Then, D = 0

P-4: If all the elements of any row (or column) be multiplied by the same number, then the value of determinant is multiplied by that number.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$. Then $D' = kD$

P-5: If each element of any row (or column) can be expressed as a sum of two terms, then the determinant can be expressed as the sum of the determinants.

e.g.,

$$\begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

P-6: The value of a determinant is not altered by adding to the elements of any row (or column) the same multiples of the corresponding elements of any other row (or column)

e.g., $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D' = \begin{vmatrix} a_1 + ma_2 & b_1 + mb_2 & c_1 + mc_2 \\ a_2 & b_2 & c_2 \\ a_3 - na_1 & b_3 - nb_1 & c_3 - nc_1 \end{vmatrix}$. Then

$D' = D$

Note: It should be noted that while applying **P-6** at least one row (or column) must remain unchanged.

P-7 : If all elements below leading diagonal or above leading diagonal or except leading diagonal elements are zero then the value of the determinant equal to multiplied of all leading diagonal elements.

e.g., $\begin{vmatrix} a_1 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{vmatrix} = a_1 b_2 c_3$

P-8: If a determinant D becomes zero on putting $x = \alpha$, then we say that $(x - \alpha)$ is factor of determinant.

e.g., if $D = \begin{vmatrix} x & 5 & 2 \\ x^2 & 9 & 4 \\ x^3 & 16 & 8 \end{vmatrix}$. At $x = 2$, $D = 0$ (because C_1 and C_2 are identical at $x = 2$)

Hence $(x - 2)$ is a factor of D.

Note: It should be noted that while applying operations on determinants then at least one row (or column) must remain unchanged or Maximum number of operations = order or determinant −1

It should be noted that if the row (or column) which is changed by multiplied a non-zero number, then the determinant will be divided by that number.