Differentiation and Integration of Determinants.

(1) **Differentiation of a determinant:**

(i) Let $\Delta(x)$ be a determinant of order two. If we write $\Delta(x) = |C_1 \quad C_2|$, where C_1 and C_2 denote the 1st and 2nd columns, then

$\Delta'(x) = |C'_1 \quad C_2| + |C_1 \quad C'_2|$

Where C'_i denotes the column which contains the derivative of all the functions in the i^{th} column C_i.

In a similar fashion, if we write $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R'_1 \\ R_2 \end{vmatrix} + \begin{vmatrix} R_1 \\ R'_2 \end{vmatrix}$

(ii) Let $\Delta(x)$ be a determinant of order three. If we write $\Delta(x) = |C_1 \quad C_2 \quad C_3|$, then

$\Delta'(x) = |C'_1 \quad C_2 \quad C_3| + |C_1 \quad C'_2 \quad C_3| + |C_1 \quad C_2 \quad C'_3|$

and similarly if we consider $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R'_1 \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R'_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R'_3 \end{vmatrix}$

(iii) If only one row (or column) consists functions of x and other rows (or columns) are constant, viz.

Let $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$,

Then $\Delta'(x) = \begin{vmatrix} f'_1(x) & f'_2(x) & f'_3(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ and in general $\Delta^n(x) = \begin{vmatrix} f_1^n(x) & f_2^n(x) & f_3^n(x) \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Where n is any positive integer and $f^n(x)$ denotes the n^{th} derivative of $f(x)$.