Adjoint of a Square Matrix.

Let $A = [a_{ij}]$ be a square matrix of order *n* and let C_{ij} be cofactor of a_{ij} in A. Then the transpose of the matrix of cofactors of elements of *A* is called the adjoint of *A* and is denoted by *adj* A

Thus, $adj\ A = [C_{ij}]^T \Rightarrow (adj\ A)_{ij} = C_{ji} =$ cofactor of a_{ji} in *A*.

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $adj\ A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$;

Where C_{ij} denotes the cofactor of a_{ij} in *A*.

Example: $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, C_{11} = s, C_{12} = -r, C_{21} = -q, C_{22} = p$

$$\therefore adj\ A = \begin{bmatrix} s & -r \\ -q & p \end{bmatrix}^T = \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$$

Note: The adjoint of a square matrix of order 2 can be easily obtained by interchanging the diagonal elements and changing signs of off diagonal elements.