Elementary Operations for Matrices

Elementary operations for matrices play a crucial role in finding the inverse or solving linear systems. They may also be used for other calculations. On this page, we will discuss these type of operations. Before we define an elementary operation, recall that to an nxm matrix *A*, we can associate n rows and m columns. For example, consider the matrix

$$A = \begin{pmatrix} 0 & 1 & -1 & 3 \\ 0 & 2 & 3 & 1 \\ -1 & 0 & 2 & -3 \end{pmatrix}.$$

Its rows are

$$\begin{pmatrix} 0 & 1 & -1 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 2 & -3 \end{pmatrix}.$$

Its columns are

$$\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -3 \end{pmatrix}.$$

Let us consider the matrix transpose of *A*

$$A^T = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 3 & 2 \\ 3 & 1 & -3 \end{pmatrix}.$$

Its rows are

$$\begin{pmatrix} 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 1 & -3 \end{pmatrix}.$$

As we can see, the transpose of the columns of A are the rows of A^T. So the transpose operation interchanges the rows and the columns of a matrix. Therefore many techniques which are developed for rows may be easily translated to columns via the transpose operation. Thus, we will only discuss elementary row operations, but the reader may easily adapt these to columns.

Elementary Row Operations.

1.

Interchange two rows.

2.

Multiply a row with a nonzero number.

3.

Add a row to another one multiplied by a number.

Definition. Two matrices are **row equivalent** if and only if one may be obtained from the other one via elementary row operations.