Rotation matrix

In linear algebra, a **rotation matrix** is a matrix that is used to perform a rotation in Euclidean space. For example, using the convention below, the matrix

rotates points in the *xy*-plane counterclockwise through an angle θ about the origin of a two-dimensional Cartesian coordinate system. To perform the rotation on a plane point with standard coordinates $\mathbf{v}$ = (x,y), it should be written as column vector, and multiplied by the matrix R:

The examples in this article apply to *active rotations* of vectors *counterclockwise* in a *right-handed coordinate system* (y counterclockwise from x) by *pre-multiplication* (R on the left). If any one of these is changed (such as rotating axes instead of vectors, a *passive transformation*), then the inverse of the example matrix should be used, which coincides with its transpose.

Since matrix multiplication has no effect on the zero vector (the coordinates of the origin), rotation matrices describe rotations about the origin. Rotation matrices provide an algebraic description of such rotations, and are used extensively for computations in geometry, physics, and computer graphics. In some literature, the term *rotation* is generalized to include improper rotations, characterized by orthogonal matrices with determinant −1 (instead of +1). These combine *proper* rotations with *reflections* (which invert orientation). In other cases, where reflections are not being considered, the label *proper* may be dropped. The latter convention is followed in this article.

Rotation matrices are square matrices, with real entries. More specifically, they can be characterized as orthogonal matrices with determinant 1; that is, a square matrix R is a rotation matrix if and only if $R^{\mathrm{T}} = R^{-1}$ and $\det R = 1$. The set of all orthogonal matrices of size n with determinant +1 forms a group known as the special orthogonal group $\mathrm{SO}(n)$, one example of which is the rotation group $\mathrm{SO}(3)$. The set of all orthogonal matrices of size n with determinant +1 or −1 forms the (general) orthogonal group $\mathrm{O}(n)$.