Types of Matrices.

(1) **Row matrix**: A matrix is said to be a row matrix or row vector if it has only one row and any number of columns. *Example*: [5 0 3] is a row matrix of order 1× 3 and [2] is a row matrix of order 1×1.

(2) **Column matrix:** A matrix is said to be a column matrix or column vector if it has only one column and any number of rows. *Example*: $\begin{bmatrix} 2 \\ 3 \\ -6 \end{bmatrix}$ is a column matrix of order 3×1 and [2] is a column matrix of order 1×1. Observe that [2] is both a row matrix as well as a column matrix.

(3) **Singleton matrix**: If in a matrix there is only one element then it is called singleton matrix. Thus, $A = [aij]_{m\times n}$ is a singleton matrix if $m = n = 1$ *Example*:[2], [3], [a], [–3] are singleton matrices.

(4) **Null or zero matrix:** If in a matrix all the elements are zero then it is called a zero matrix and it is generally denoted by *O*. Thus $A = [a_{ij}]_{m\times n}$ is a zero matrix if $a_{ij} = 0$ for all *i* and *j*.

Example: $[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, [0\ \ 0]$ are all zero matrices, but of different orders.

(5) **Square matrix**: If number of rows and number of columns in a matrix are equal, then it is called a square matrix. Thus $A = [a_{ij}]_{m\times n}$ is a square matrix if $m = n$. *Example*: $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3×3

(i) If $m \neq n$ then matrix is called a rectangular matrix.

(ii) The elements of a square matrix A for which $i = j, i.e.\ a_{11}, a_{22}, a_{33}, \ldots. a_{nn}$ are called diagonal elements and the line joining these elements is called the principal diagonal or leading diagonal of matrix *A*.

(iii) **Trace of a matrix:** The sum of diagonal elements of a square matrix. *A* is called the trace of matrix A , which is denoted by tr A. $tr\ A = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \dots a_{nn}$

Properties of trace of a matrix: Let $A = [a_{ii}]_{n\times n}$ and $B = [b_{ij}]_{n\times n}$ and λ be a scalar

(i) $tr(\lambda A) = \lambda\, tr(A)$

(ii) $tr(A - B) = tr(A) - tr(B)$

(iii) $tr(AB) = tr(BA)$

(iv) $tr(A) = tr(A')$ or $tr\ (A^T)$

(v) $tr(I_n) = n$

(vi) *tr* (0)= 0

(vii) $tr(AB) \neq tr\, A\, .\, tr\, B$

(6) **Diagonal matrix:** If all elements except the principal diagonal in a square matrix are zero, it is called a diagonal matrix. Thus a square matrix $A = [a_{ij}]$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$.

Example: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a diagonal matrix of order 3×3, which can be denoted by diag [2, 3, and 4]

Note: No element of principal diagonal in a diagonal matrix is zero.

- Number of zeros in a diagonal matrix is given by $n^2 - n$ where *n* is the order of the matrix.
- A diagonal matrix of order $n \times n$ having $d_1, d_2, \dots\dots, d_n$ as diagonal elements is denoted by $\text{diag}\ [d_1, d_2, \dots, d_n]$.

(7) **Identity matrix**: A square matrix in which elements in the main diagonal are all '1' and rest are all zero is called an identity matrix or unit matrix. Thus, the square matrix $A = [a_{ij}]$ is an identity matrix, if $a_{ij} = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ if } i \neq j \end{cases}$

We denote the identity matrix of order *n* by I_n.

Example: [1], $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 1, 2 and 3 respectively.

(8) **Scalar matrix** : A square matrix whose all non-diagonal elements are zero and diagonal elements are equal is called a scalar matrix. Thus, if $A = [a_{ij}]$ is a square matrix and $a_{ij} = \begin{cases} \alpha, \text{if } i = j \\ 0, \text{ if } i \neq j \end{cases}$, then *A* is a scalar matrix.

Example: [2], $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ are scalar matrices of order 1, 2 and 3 respectively.

Note: Unit matrix and null square matrices are also scalar matrices.

(9) **Triangular Matrix**: A square matrix $[a_{ij}]$ is said to be triangular matrix if each element above or below the principal diagonal is zero. It is of two types

(i) **Upper Triangular matrix:** A square matrix $[a_{ij}]$ is called the upper triangular matrix, if $a_{ij} = 0$ when $i > j$.

Example: $\begin{bmatrix} 3 & 1 & 2 \\ 0 & 4 & 3 \\ 0 & 0 & 6 \end{bmatrix}$ is an upper triangular matrix of order 3×3.

(ii) **Lower Triangular matrix:** A square matrix $[a_{ij}]$ is called the lower triangular matrix, if $a_{ij} = 0$ when *i* < *j*.

Example: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 2 \end{bmatrix}$ is a lower triangular matrix of order 3×3.

Note: Minimum number of zeros in a triangular matrix is given by $\frac{n(n-1)}{2}$ where n is order of matrix.

- Diagonal matrix is both upper and lower triangular.
- A triangular matrix $a = [a_{ij}]_{n \times n}$ is called strictly triangular if $a_{ij} = 0$ for $1 \le i \le n$