Multiplication of Matrices.

Two matrices A and B are conformable for the product AB if the number of columns in A (pre-multiplier) is same as the number of rows in *B* (post multiplier).Thus, if $A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{n\times p}$ are two matrices of order *m*×*n* and $n\times p$ respectively, then their product *AB* is of order $m\times p$ and is defined as $(AB)_{ij}=\sum_{r=1}^{n} a_{ir}b_{rj}$

$= [a_{i1}a_{i2}...a_{in}]\begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{nj} \end{bmatrix}$ = (i^{th} row of A)(j^{th} column of *B*) .....(i), where *i*=1, 2, ..., *m* and *j*=1, 2, ...*p*

Now we define the product of a row matrix and a column matrix.

Let $A=[a_1 a_2 .... a_n]$ be a row matrix and $B=\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ be a column matrix.

Then $AB=[a_1b_1+a_2b_2+....+a_nb_n]$ ...(ii). Thus, from (i),

$(AB)_{ij}=$ Sum of the product of elements of i^{th} row of *A* with the corresponding elements of j^{th} column of *B*.

Properties of matrix multiplication

If *A*,*B* and *C* are three matrices such that their product is defined, then

(i) $AB \neq BA$	(Generally not commutative)
(ii) $(AB)C = A(BC)$	(Associative Law)
(iii) $IA = A = AI$	Where I is identity matrix for matrix multiplication
(iv) $A(B+C)=AB+AC$	(Distributive law)
(v) If $AB = AC \not\Rightarrow B = C$	(Cancellation law is not applicable)

(vi) If *AB*= 0 It does not mean that *A*= 0 or *B* = 0, again product of two non zero matrix may be a zero matrix.

Note: If *A* and *B* are two matrices such that *AB* exists, then *BA* may or may not exist.

❑ The multiplication of two triangular matrices is a triangular matrix.

❑ The multiplication of two diagonal matrices is also a diagonal matrix and diag $(a_1, a_2, ... a_n) \times \text{diag } (b_1, b_2, ... b_n) = \text{diag } (a_1b_1, a_2b_2, ... a_nb_n)$

❑ The multiplication of two scalar matrices is also a scalar matrix.

❑ If A and B are two matrices of the same order, then

(i) $(A+B)^2 = A^2 + B^2 + AB + BA$

(ii) $(A-B^2) = A^2 + B^2 - AB - BA$

(iii) $(A-B)(A+B) = A^2 - B^2 + AB - BA$

(iv) $(A+B)(A-B) = A^2 - B^2 - AB + BA$

(v) $A(-B) = (-A)B = -(AB)$