

## Definition.

A binary operation on a non-empty set  $A$  is a mapping which associates with each ordered pair  $(a, b)$  of elements of  $A$ , a uniquely defined element  $c \in A$ . This is a mapping from the product set  $A \times A$  to  $A$ . Symbolically, a map:  $A \times A \rightarrow A$ , is called a binary operation on the set  $A$ .

The image of the element  $(a, b) \in A \times A$  is denoted by  $a * b$ . If a set  $A$  is closed with respect to the composition, then we say that  $*$  is a binary operation on the set  $A$ .

Let,  $a \in \mathbb{N}, b \in \mathbb{N} \Rightarrow a + b \in \mathbb{N}$  for all  $a, b \in \mathbb{N}$ .

Multiplication on  $\mathbb{N}$  is also a binary operation, since  $a \in \mathbb{N}, b \in \mathbb{N} \Rightarrow a \times b \in \mathbb{N}$  for all  $a, b \in \mathbb{N}$

But subtraction on  $\mathbb{N}$  is not a binary operation, since  $3 \in \mathbb{N}, 5 \in \mathbb{N}$  but  $3 - 5 = -2 \notin \mathbb{N}$ .

Note: It is obvious that addition as well as multiplication are binary operations on each one of the sets  $\mathbb{Z}$  (of integer),  $\mathbb{Q}$  (of rational number),  $\mathbb{R}$  (of real number) and  $\mathbb{C}$  (of all complex number).

□ Subtraction is a binary operation on each of the sets  $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$  and  $\mathbb{C}$ . But it is not binary operation on  $\mathbb{N}$ .

□ Division is not a binary operation on any of sets  $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$  and  $\mathbb{C}$ .