

Identity and Inverse elements

(1) **Identity element:** Let $*$ be a binary operation on a set S . An element $e \in S$ is said to be an identity element for the binary operation $*$ if $a * e = a = e * a$ for all $a \in S$.

For addition on \mathbb{Z} , 0 is the identity element, since $a + 0 = a = 0 + a$ for all $a \in \mathbb{Z}$.

For multiplication on \mathbb{R} , 1 is the identity element, since $1 \times a = a = a \times 1$ for all $a \in \mathbb{R}$.

(2) **Inversible element for a binary operation with identity:** An element a of a set A is said to be invertible for a binary operation $*$ with identity e if $\exists b \in A$ such that $a * b = e = b * a$.

Also, then b is said to be an inverse of a and is denoted by a^{-1} . The invertible elements in A are also called the units in A . The identity element is always invertible and is its own inverse, since $e * e = e * e = e$. Thus $e^{-1} = e$.