Composition Table

A binary operation on a finite set can be completely described by means of a table known as a composition table. Let $S = \{a_1, a_2, \ldots.., a_n\}$ be a finite set and * be a binary operation on S. Then the composition table for * is constructed in the manner indicated below.

We write the elements a_1, a_2, ...,a_n of the set S in the top horizontal row and the left vertical column in the same order. Then we put down the element $a_i * a_j$ at the intersection of the row headed by a_i ($1 \le i \le n$) and the column headed by $a_j (1 \le j \le n)$ to get the following table.

*	a_1	a_2	.....	a_i	.....	a_j	.....	a_n
a_1	$a_1 * a_1$	$a_1 * a_2$	.....	$a_1 * a_i$	.....	$a_1 * a_j$	.....	$a_1 * a_n$
a_2	$a_2 * a_1$	$a_2 * a_2$	.....	$a_2 * a_i$	.....	$a_2 * a_j$	.....	$a_2 * a_n$
$\vdots$								
a_i	$a_i * a_1$	$a_i * a_2$	.....	$a_i * a_i$	.....	$a_i * a_j$	.....	$a_i * a_n$
$\vdots$								
a_j	$a_j * a_1$	$a_j * a_2$	.....	$a_j * a_i$	.....	$a_j * a_j$	.....	$a_j * a_n$
$\vdots$								
a_n	$a_n * a_1$	$a_n * a_2$	.....	$a_n * a_i$	.....	$a_n * a_j$	.....	$a_n * a_n$

From the composition table we infer the following results:

(1) If all the entries of the table are elements of set S and each element of S appears once and only once in each row and in each column, then the operation is a binary operation. Sometimes we also say that the binary operation is well defined which means that the operation * associates each pair of elements of S to a unique element of S, i.e. S is closed under the operation *.

(2) If the entries in the table are symmetric with respect to the diagonal which starts at the upper left corner of the table and terminates at the lower right corner, we say that the binary operation is commutative on S, otherwise it is said to be not commutative on S.

(3) If the row headed by an element say a_j, coincides with the row at the top and the column headed by a_j coincides with the column on extreme left, then a_j is the identity element for the binary operation * on S.

(4) If each row except the topmost row or each column except the left most column contains the identity element then every element of S is invertible with respect to *. To find the inverse of an element say a_j, we consider row (or column) headed by a_i. Then we determine the position of identity element e in this row (or column). If e appears in the column (or row) headed by a_j, then a_i and a_j are inverse of each other.

It should be noted that the composition table is helpless to determine associativity of the binary operation. This has to be verified for each possible trial.