Distance Formula.

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(PR)^2 + (QR)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note: \Box The distance of a point $M(x_0, y_0)$ from origin O(0, 0)

$$OM = \sqrt{(x_0^2 + y_0^2)}.$$

If distance between two points is given then use \pm sign.

When the line PQ is parallel to the y-axis, the abscissa of point P and Q will be equal i.e, $x_1 = x_2$; $\therefore PQ = |y_2 - y_1|$

When the segment PQ is parallel to the x-axis, the ordinate of the points P and Q will be equal i.e., $y_1 = y_2$. Therefore $PQ = |x_2 - x_1|$

(1) **Distance between two points in polar co-ordinates**: Let O be the pole and OX be the initial line. Let P and Q be two given points whose polar co-ordinates are (r_1, θ_1) and (r_2, θ_2) respectively.

Then
$$OP = r_1, OQ = r_2$$

 $\angle POX = \theta_1 \text{ and } \angle QOX = \theta_2$
Then $\angle POQ = (\theta_1 - \theta_2)$

In $\triangle POQ$, from cosine rule $\cos(\theta_1 - \theta_2) = \frac{(OP)^2 + (OQ)^2 - (PQ)^2}{2OP \cdot OQ}$

$$\therefore (PQ)^2 = r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)$$

$$\therefore PQ = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

Note: Always taking θ_1 and θ_2 in radians.



