## Geometrical Conditions.

(1) Properties of triangles
(i) In any triangle $A B C, A B+B C>A C$ and $|A B-B C|<A C$.
(ii) The $\triangle A B C$ is equilateral $\Leftrightarrow A B=B C=C A$.
(iii) The $\triangle A B C$ is a right angled triangle $\Leftrightarrow A B^{2}=A C^{2}+B C^{2}$ or $A C^{2}=A B^{2}+B C^{2}$ or $B C^{2}=A B^{2}+A C^{2}$.
(iv) The $\triangle A B C$ is isosceles $\Leftrightarrow A B=B C$ or $B C=C A$ or $A B=A C$.

## (2) Properties of quadrilaterals

(i) The quadrilateral $A B C D$ is a parallelogram if and only if

(a) $A B=D C, A D=B C$, or (b) the middle points of BD and AC are the same,

In a parallelogram diagonals AC and BD are not equal and $\theta \neq \frac{\pi}{2}$.
(ii) The quadrilateral $A B C D$ is a rectangle if and only if
(a) $A B=C D, A D=B C$ and $A C^{2}=A B^{2}+B C^{2}$ or, (b) $A B=C D, A D=B C, A C=B D$ or, (c) the middle points of $A C$ and BD are the same and $\mathrm{AC}=\mathrm{BD} .(\theta \neq \pi / 2)$

(iii) The quadrilateral $A B C D$ is a rhombus (but not a square) if and only if (a) $A B=B C=C D=D A$ and $A C \neq B D$ or, (b) the middle points of $A C$ and $B D$ are the same and $A B=A D$ but $A C \neq B D .(\theta=\pi / 2)$
(iv) The quadrilateral $A B C D$ is a square if and only if

(a) $A B=B C=C D=D A$ and $A C=B D$ or (b) the middle points of AC and BD are the same and $A C=B D,(\theta=\pi / 2), A B=A D$.


Note: Diagonals of square, rhombus, rectangle and parallelogram always bisect each other. $\square$ Diagonals of rhombus and square bisect each other at right angle.
$\square$ Four given points are collinear, if area of quadrilateral is zero.

