Geometrical Conditions.

(1) Properties of triangles

- (i) In any triangle ABC, AB + BC > AC and |AB BC| < AC.
- (ii) The $\triangle ABC$ is equilateral $\Leftrightarrow AB = BC = CA$.
- (iii) The $\triangle ABC$ is a right angled triangle $\Leftrightarrow AB^2 = AC^2 + BC^2$ or $AC^2 = AB^2 + BC^2$ or
- $BC^2 = AB^2 + AC^2.$
- (iv) The $\triangle ABC$ is isosceles $\Leftrightarrow AB = BC$ or BC = CA or AB = AC.

(2) Properties of quadrilaterals

- (i) The quadrilateral ABCD is a parallelogram if and only if
- (a) AB = DC, AD = BC, or (b) the middle points of BD and AC are the same,
- In a parallelogram diagonalsAC and BD are not equal and $\theta \neq \frac{\pi}{2}$.
- (ii) The quadrilateral ABCD is a rectangle if and only if

(a) AB = CD, AD = BC and $AC^2 = AB^2 + BC^2$ or, (b) AB = CD, AD = BC, AC = BD or, (c) the middle points of AC and BD are the same and AC=BD. ($\theta \neq \pi/2$)

(iii) The quadrilateral *ABCD* is a rhombus (but not a square) if and only if (a) AB = BC = CD = DA and $AC \neq BD$ or, (b) the middle points of AC and BD are the same and AB = AD but $AC \neq BD$. ($\theta = \pi/2$)

(iv) The quadrilateral *ABCD* is a square if and only if (a) AB = BC = CD = DA and AC = BD or (b) the middle points of AC and BD are the same and AC = BD, $(\theta = \pi / 2)$, AB = AD.









Note: Diagonals of square, rhombus, rectangle and parallelogram always bisect each other.
Diagonals of rhombus and square bisect each other at right angle.
Four given points are collinear, if area of quadrilateral is zero.