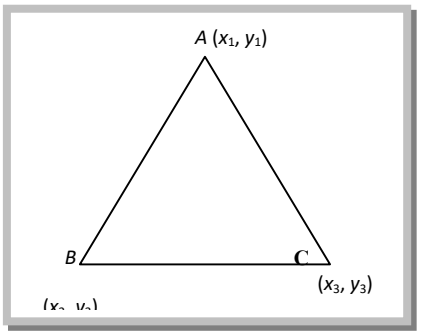
Area of some Geometrical figures.

(1) **Area of a triangle**: The area of a triangle ABC with vertices $A(x_1,y_1);B(x_2,y_2)$ and $C(x_3,y_3)$.

The area of triangle ABC is denoted by 'Δ'and is given as



$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\left|(x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

In equilateral triangle

(i) Having sides a, area is $\frac{\sqrt{3}}{4}a^2$.

(ii) Having length of perpendicular as 'p' area is $\frac{(p^2)}{\sqrt{3}}$.

Note: If a triangle has polar co-ordinates $(r_1,\theta_1),(r_2,\theta_2)$ and (r_3,θ_3) then its area

$$\Delta = \frac{1}{2}[r_1r_2\sin(\theta_2-\theta_1)+r_2r_3\sin(\theta_3-\theta_2)+r_3r_1\sin(\theta_1-\theta_3)]$$

If area is a rational number. Then the triangle cannot be equilateral.

(2)**Collinear points:** Three points $A(x_1,y_1);B(x_2,y_2);C(x_3,y_3)$ are collinear. If area of triangle is zero,

i.e., (i) $\Delta = 0 \Rightarrow \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

(ii) $AB + BC = AC$ or $AC + BC = AB$ or $AC + AB = BC$

(3)**Area of a quadrilateral**: If $(x_1,y_1);(x_2,y_2);(x_3,y_3)$ and (x_4,y_4) are vertices of a quadrilateral, then its Area $= \frac{1}{2}[(x_1y_2 - x_2y_1)+(x_2y_3 - x_3y_2)+(x_3y_4 - x_4y_3)+(x_4y_1 - x_1y_4)]$

Note: If two opposite vertex of rectangle are (x_1, y_1) and (x_2, y_2), then its area is $|(y_2 - y_1)(x_2 - x_1)|$.

It two opposite vertex of a square are $A(x_1, y_1)$ and $C(x_2, y_2)$, then its area is

$$= \frac{1}{2} AC^2 = \frac{1}{2}[(x_2 - x_1)^2 + (y_2 - y_1)^2]$$

(4) **Area of polygon**: The area of polygon whose vertices are $(x_1, y_1); (x_2, y_2); (x_3, y_3); \ldots (x_n, y_n)$ is

$$= \frac{1}{2} | \{(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + \ldots + (x_n y_1 - x_1 y_n)\} |$$

or**Stair method:** Repeat first co-ordinates one time in last for down arrow use positive sign and for up arrow use negative sign.

$$\therefore \quad \text{Area of polygon} = \frac{1}{2} \left| \begin{matrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ : & : \\ : & : \\ x_n & y_n \\ x_1 & y_1 \end{matrix} \right| = \frac{1}{2} | \{(x_1 y_2 + x_2 y_3 + \ldots + x_n y_1) - (y_1 x_2 + y_2 x_3 + \ldots + y_n x_1)\} |$$