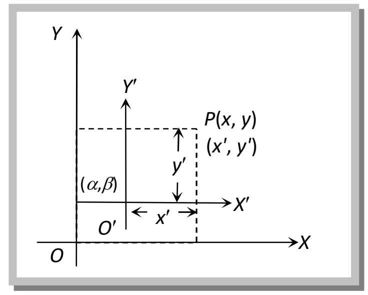
Transformation of Axes.

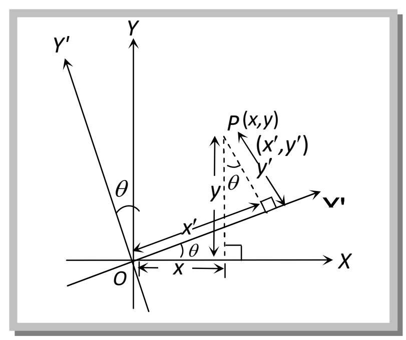
(1) **Shifting of origin without rotation of axes**: Let $P \equiv (x, y)$ with respect to axes OX and OY. Let $O' \equiv (\alpha, \beta)$ with respect to axes OX and OY and let $P \equiv (x', y')$ with respect to axes O'X' and O'Y', where OX and O'X' are parallel and OY and O'Y' are parallel.



Then $x = x' + \alpha, y = y' + \beta$ or $x' = x - \alpha, y' = y - \beta$

Thus if origin is shifted to point (α, β) without rotation of axes, then new equation of curve can be obtained by putting $x + \alpha$ in place of x and $y + \beta$ in place of y.

(2)**Rotation of axes without changing the origin:**Let O be the origin. Let $P \equiv (x, y)$ with respect to axes OX and OY and let $P \equiv (x', y')$ with respect to axes OX′ and OY′ where $\angle X'OX = \angle YOY' = \theta$



Then $x = x' \cos\theta - y' \sin\theta$

$y = x' \sin\theta + y' \cos\theta$

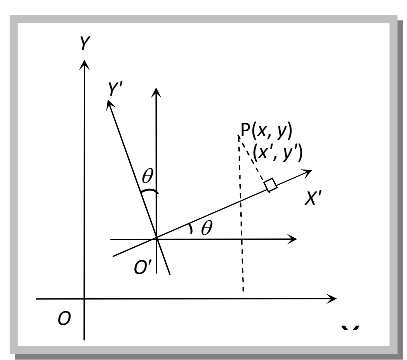
and $x' = x \cos\theta + y \sin\theta$

$y' = -x \sin\theta + y \cos\theta$

The above relation between (x, y) and (x', y') can be easily obtained with the help of following table

	$x \downarrow$	$y \downarrow$
$x' \rightarrow$	$\cos\theta$	$\sin\theta$
$y' \rightarrow$	$-\sin\theta$	$\cos\theta$

(3) **Change of origin and rotation of axes:** If origin is changed to $O'(\alpha, \beta)$ and axes are rotated about the new origin O' by an angle θ in the anticlock-



wise sense such that the new co-ordinates of $P(x, y)$ become (x', y') then the equations of transformation will be $x = \alpha + x'\cos\theta - y'\sin\theta$ and $y = \beta + x'\sin\theta + y'\cos\theta$

(4)**Reflection (Image of a point):** Let (x, y) be any point, then its image with respect to

(i) x axis $\Rightarrow (x, -y)$ (ii) y-axis $\Rightarrow (-x, y)$ (iii) origin $\Rightarrow (-x, -y)$ (iv) line $y = x \Rightarrow (y, x)$