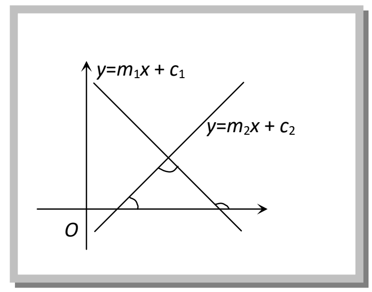
Angle between Two non-parallel Lines.

Let θ be the angle between the lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$.

and intersecting at A.



Where, $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

$\because \quad \alpha = \theta + \beta \Rightarrow \theta = \alpha - \beta$

$\Rightarrow \quad \tan\theta = \left|\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right|$

$\therefore \quad \theta = \tan^{-1}\left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$.

(1)**Angle between two straight lines when their equations are given:**The angle θ between the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ is given by, $\tan\theta = \left|\dfrac{a_2 b_1 - a_1 b_2}{a_1 a_2 + b_1 b_2}\right|$.

(i) **Condition for the lines to be parallel:**If the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel then, $m_1 = m_2 \Rightarrow \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} \Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$.

(ii) **Condition for the lines to be perpendicular:** If the lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are perpendicular then, $m_1 m_2 = -1 \Rightarrow \dfrac{a_1}{b_1} \times \dfrac{a_2}{b_2} = -1 \Rightarrow a_1 a_2 + b_1 b_2 = 0$.

(iii) **Conditions for two lines to be coincident, parallel, perpendicular and intersecting:**Two lines $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are,

(a) Coincident, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

(b) Parallel, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

(c) Intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(d) Perpendicular, if $a_1 a_2 + b_1 b_2 = 0$