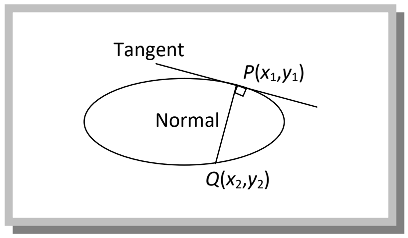
Equations of Normal in Different forms.

(1) **Point form:** The equation of the normal at (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2.$$



(2) **Parametric form:** The equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $(a\cos\phi, b\sin\phi)$ is

$$ax\sec\phi - by\operatorname{cosec}\phi = a^2-b^2.$$

(3) **Slope form:** If m is the slope of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the equation of normal is

$$y=mx\pm\frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$$

The coordinates of the point of contact are $\left(\frac{\pm a^2}{\sqrt{a^2+b^2m^2}}, \frac{\pm mb^2}{\sqrt{a^2+b^2m^2}}\right)$

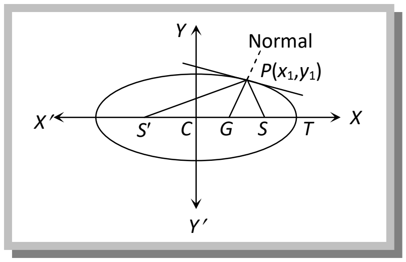
Note: If $y=mx+c$ is the normal of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then condition of normality is $c^2=\frac{m^2(a^2-b^2)^2}{(a^2+b^2m^2)}$.

The straight line $lx+my+n=0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\left(\frac{a^2-b^2}{n^2}\right)^2$.

Four normals can be drawn from a point to an ellipse.

Important Tips

☞ If S be the focus and G be the point where the normal at P meets the axis of an ellipse, then $SG = e.SP$, and the tangent and normal at P bisect the external and internal angles between the focal distances of P.



☞ Any point P of an ellipse is joined to the extremities of the major axis then the portion of a directrix intercepted by them subtends a right angle at the corresponding focus.

☞ With a given point and line as focus and directrix, a series of ellipse can be described. The locus of the extremities of their minor axis is a parabola.

☞ The equations to the normals at the end of the latera recta and that each passes through an end of the minor axis, if $e^4 + e^2 + 1 = 0$

☞ If two concentric ellipse be such that the foci of one be on the other and if e and e′ be their eccentricities. Then the angle between their axes is $\cos^{-1}\sqrt{\frac{e^2 + e'^2 - 1}{ee'}}$.