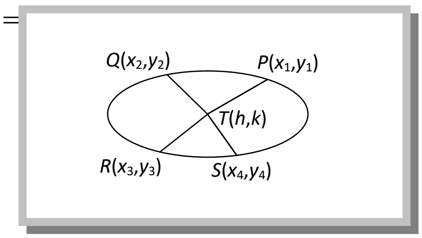
Properties of Eccentric angles of the Co-normal points.

(1) The sum of the eccentric angles of the co-normal points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to odd multiple of π.

(2) If α, β, γ are the eccentric angles of three points on the ellipse, the normals at which are concurrent, then $\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\alpha)=0$.

(3)**Co-normal points lie on a fixed curve:** Let $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ be co-normal points, then PQRS lie on the curve $(a^2-b^2)xy+b^2kx-a^2hy=$

This curve is called Apollonian rectangular hyperbola.



Note: The feet of the normals from any fixed point to the ellipse lie at the intersections of the apollonian rectangular hyperbola with the ellipse.

Important Tips

☞ The area of the triangle formed by the three points, on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose eccentric angles are θ, ϕ and ψ is $2ab\sin\left(\frac{\phi-\psi}{2}\right)\sin\left(\frac{\psi-\theta}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$.

☞ The eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by $2\cot w=\frac{e^2\sin 2\theta}{\sqrt{(1-e^2)}}$, where w is one of the angles between the normals at the points whose eccentric angles are θ and $\frac{\pi}{2}+\theta$.