Equation of the Chord joining two points on an Ellipse.

Let $P(a\cos\theta, b\sin\theta)$; $Q(a\cos\phi, b\sin\phi)$ be any two points of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Then, the equation of the chord joining these two points is $y - b\sin\theta = \frac{b\sin\phi - b\sin\theta}{a\cos\phi - a\cos\theta}(x - a\cos\theta)$

Thus, the equation of the chord joining two points having eccentric angles θ and ϕ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right)=\cos\left(\frac{\theta-\phi}{2}\right)$

Note: If the chord joining two points whose eccentric angles are α and β cut the major axis of an ellipse at a distance 'c' from the center, then $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{c-a}{c+a}$.

If α and β be the eccentric angles of the extremities of a focal chord of an ellipse of eccentricity e, then $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\frac{1\mp e}{1\pm e}=0$.