

## Definition.

An ellipse is the locus of a point which moves in such a way that its distance from a fixed point is in constant ratio ( $< 1$ ) to its distance from a fixed line. The fixed point is called the **focus** and fixed line is called the **directrix** and the constant ratio is called the **eccentricity** of the ellipse, denoted by ( $e$ ).

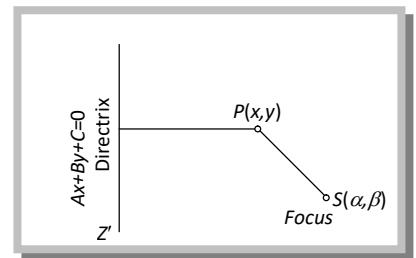
In other words, we can say an ellipse is the locus of a point which moves in a plane so that the sum of its distances from two fixed points is constant and is more than the distance between the two fixed points.

Let  $S(\alpha, \beta)$  is the focus,  $ZZ'$  is the directrix and  $P$  is any point on the ellipse. Then by definition,

$$\frac{SP}{PM} = e \Rightarrow SP = e \cdot PM$$

$$\sqrt{(x - \alpha)^2 + (y - \beta)^2} = e \frac{Ax + By + C}{\sqrt{A^2 + B^2}}$$

Squaring both sides,  $(A^2 + B^2)[(x - \alpha)^2 + (y - \beta)^2] = e^2(Ax + By + C)^2$



Note: The condition for second degree equation in  $x$  and  $y$  to represent an ellipse is that  $h^2 - ab < 0$  and  $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$