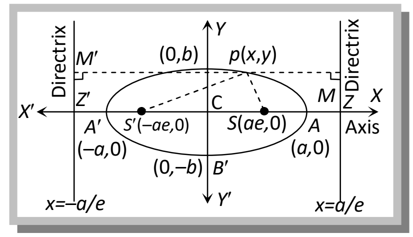
# Standard equation of the Ellipse.



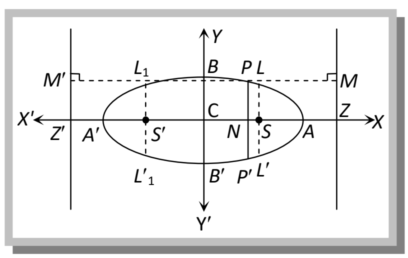

Let S be the focus, ZM be the directrix of the ellipse and $P(x,y)$ is any point on the ellipse, then by definition $\frac{SP}{PM}=e \Rightarrow (SP)^2=e^2(PM)^2$

$$(x-ae)^2+(y-0)^2=e^2\left(\frac{a}{e}-x\right)^2 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2=a^2(1-e^2)$

Since $e<1$, therefore $a^2(1-e^2)<a^2 \Rightarrow b^2<a^2$. Some terms related to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$:

(1) **Centre:** The point which bisects each chord of the ellipse passing through it, is called center $(0,0)$ denoted by C.




(2) **Major and minor axes:** The diameter through the foci, is called the major axis and the diameter bisecting it at right angles is called the minor axis. The major and minor axes are together called principal axes.

Length of the major axis $AA'=2a$, Length of the minor axis $BB'=2b$

The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is symmetrical about both the axes.

(3) **Vertices:** The extremities of the major axis of an ellipse are called vertices.

The coordinates of vertices A and $A'$ are (a, 0) and (–a, 0) respectively.

(4) **Foci:** S and $S'$ are two foci of the ellipse and their coordinates are (ae, 0) and (–ae, 0) respectively. Distance between foci $SS' = 2ae$.

(5) **Directrices:** ZM and $Z'M'$ are two directrices of the ellipse and their equations are $x = \frac{a}{e}$ and $x = -\frac{a}{e}$ respectively. Distance between directrices $ZZ' = \frac{2a}{e}$.

(6) **Eccentricity of the ellipse:** For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

we have $b^2 = a^2(1-e)^2 \Rightarrow e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{4b^2}{4a^2} = 1 - \left(\frac{2b}{2a}\right)^2$ **;** $e = \sqrt{1 - \left(\frac{\text{Minor axis}}{\text{Major axis}}\right)^2}$

This formula gives the eccentricity of the ellipse.

(7) **Ordinate and double ordinate:** Let P be a point on the ellipse and let PN be perpendicular to the major axis AA' such that PN produced meets the ellipse at $P'$. Then PN is called the ordinate of P and $PNP'$ the double ordinate of P.

If abscissa of P is h, then ordinate of P, $\frac{y^2}{b^2} = 1 - \frac{h^2}{a^2} \Rightarrow y = \frac{b}{a}\sqrt{(a^2 - h^2)}$ (For first quadrant)

And ordinate of $P'$ is $y = \frac{-b}{a}\sqrt{(a^2 - h^2)}$ (For fourth quadrant)

Hence coordinates of P and $P'$ are $\left(h, \frac{b}{a}\sqrt{(a^2 - h^2)}\right)$ and $\left(h, \frac{-b}{a}\sqrt{(a^2 - h^2)}\right)$ respectively.

(8) **Latus-rectum:** Chord through the focus and perpendicular to the major axis is called its latus rectum.

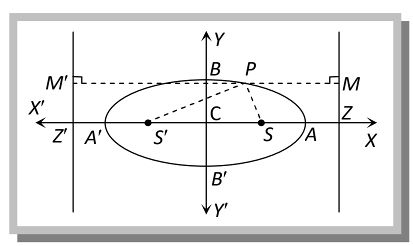
The double ordinates $LL'$ and $L_1L_1'$ are latus rectum of the ellipse.

Length of latus rectum $LL' = L_1L_1' = \frac{2b^2}{a}$ and end points of latus-rectum are

$L = \left(ae, \frac{b^2}{a}\right), L' = \left(ae, \frac{-b^2}{a}\right)$ and $L_1 = \left(-ae, \frac{b^2}{a}\right); L_1' = \left(-ae, \frac{-b^2}{a}\right)$

(9) **Focal chord:** A chord of the ellipse passing through its focus is called a focal chord.

(10) **Focal distances of a point:** The distance of a point from the focus is its focal distance. The



sum of the focal distances of any point on an ellipse is constant and equal to the length of the major axis of the ellipse.

Let $P(x_1, y_1)$ be any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$SP = ePM = e\left(\frac{a}{e} - x_1\right) = a - ex_1$ and $S'P = ePM' = e\left(\frac{a}{e} + x_1\right) = a + ex_1$

$\therefore SP + S'P = (a - ex_1) + (a + ex_1) = 2a = AA' =$ major axis.