

## Intersection of a Line and an Ellipse.

Let the ellipse be  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  .....(i) and the given line be  $y = mx + c$  .....(ii)

Eliminating  $y$  from equation (i) and (ii), then  $\frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$

$$\text{i.e., } (a^2m^2 + b^2)x^2 + 2mca^2x + a^2(c^2 - b^2) = 0$$

The above equation being a quadratic in  $x$ , its discriminant  $= 4m^2c^2a^4 - 4a^2(a^2m^2 + b^2)(c^2 - b^2)$   
 $= b^2 \{ (a^2m^2 + b^2) - c^2 \}$

Hence the line intersects the ellipse in two distinct points if  $a^2m^2 + b^2 > c^2$  in one point if  $c^2 = a^2m^2 + b^2$  and does not intersect if  $a^2m^2 + b^2 < c^2$ .