Equations of Tangent in Different forms.

(1) **Point form:** The equation of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(x_1, y_1)$is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

(2) **Slope form:** If the line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $c^2 = a^2m^2 + b^2$.

Hence, the straight line $y = mx \pm \sqrt{a^2m^2 + b^2}$ always represents the tangents to the ellipse.

Points of contact: Line $y = mx \pm \sqrt{a^2m^2 + b^2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at

$$\left(\frac{\pm a^2m}{\sqrt{a^2m^2+b^2}}, \frac{\mp b^2}{\sqrt{a^2m^2+b^2}}\right)$$

(3) **Parametric form:** The equation of tangent at any point $\phi(a\cos\phi, b\sin\phi)$ is

$$\frac{x}{a}\cos\phi+\frac{y}{b}\sin\phi=1$$

Note: The straight line $lx + my + n = 0$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $a^2l^2 + b^2m^2 = n^2$.

The line $x\cos\alpha + y\sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$ and that point of contact is $\left(\frac{a^2\cos\alpha}{p}, \frac{b^2\sin\alpha}{p}\right)$.

Two tangents can be drawn from a point to an ellipse. The two tangents are real and distinct or coincident or imaginary according as the given point lies outside, on or inside the ellipse.

The tangents at the extremities of latus-rectum of an ellipse intersect on the corresponding directrix.

Important Tips

- ☞ A circle of radius r is concentric with the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the common tangent is inclined to the major axis at an angle $\tan^{-1}\sqrt{\left(\frac{r^2-b^2}{a^2-r^2}\right)}$.
- ☞ The locus of the foot of the perpendicular drawn from centre upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(x^2+y^2)^2=a^2x^2+b^2y^2$ or $r^2=a^2\cos^2\theta+b^2\sin^2\theta$ (in polar coordinates)
- ☞ The locus of the mid points of the portion of the tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepted between the axes is $a^2y^2+b^2x^2=4x^2y^2$.
- ☞ The product of the perpendiculars from the foci to any tangent of an ellipse is equal to the square of the semi minor axis, and the feet of these perpendiculars lie on the auxiliary circle.