

Equation of Pair of Tangents $SS_1 = T^2$.

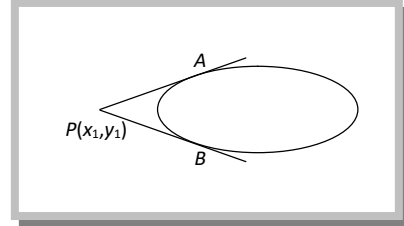
Pair of tangents: Let $P(x_1, y_1)$ be any point lying outside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let a pair of tangents PA, PB can be drawn to it from P.

Then the equation of pair of tangents PA and PB is $SS_1 = T^2$

$$\text{where } S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

$$S_1 \equiv \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 = 0$$

$$T \equiv \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$$



Director circle: The director circle is the locus of points from which perpendicular tangents are drawn to the ellipse.

Let $P(x_1, y_1)$ be any point on the locus. Equation of tangents through $P(x_1, y_1)$ is given by

$$SS_1 = T^2$$

$$\text{i.e., } \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right) \left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 \right) = \left[\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 \right]^2$$

They are perpendicular, So coeff. of x^2 + coeff. of $y^2 = 0$

$$\therefore \left(\frac{1}{a^2} + \frac{1}{b^2} \right) \left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 \right) - \left(\frac{x_1^2}{a^4} + \frac{y_1^2}{b^4} \right) = 0 \text{ or } x_1^2 + y_1^2 = a^2 + b^2$$

Hence locus of $P(x_1, y_1)$ i.e., equation of director circle is $x^2 + y^2 = a^2 + b^2$

