

Equation of the Chord joining Two points on the Hyperbola.

The equation of the chord joining the points $P(a \sec \phi_1, b \tan \phi_1)$ and $Q(a \sec \phi_2, b \tan \phi_2)$ is

$$y - b \tan \phi_1 = \frac{b \tan \phi_2 - b \tan \phi_1}{a \sec \phi_2 - a \sec \phi_1} (x - a \sec \phi_1)$$

$$\frac{x}{a} \cos\left(\frac{\phi_1 - \phi_2}{2}\right) - \frac{y}{b} \sin\left(\frac{\phi_1 + \phi_2}{2}\right) = \cos\left(\frac{\phi_1 + \phi_2}{2}\right)$$

Note: If the chord joining two points $(a \sec \theta_1, b \tan \theta_1)$ and $(a \sec \theta_2, b \tan \theta_2)$ passes through the focus

of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan \frac{\theta_1}{2} \tan \frac{\theta_2}{2} = \frac{1 - e}{1 + e}$.