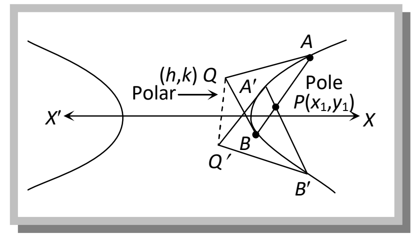
Pole and Polar.

Let P be any point inside or outside the hyperbola. If any straight line drawn through P interesects the hyperbola at A and B. Then the locus of the point of intersection of the tangents to the hyperbola at A and B is called the polar of the given point P with respect to the hyperbola and the point P is called the pole of the polar.

The equation of the required polar with (x_1, y_1) as its pole is

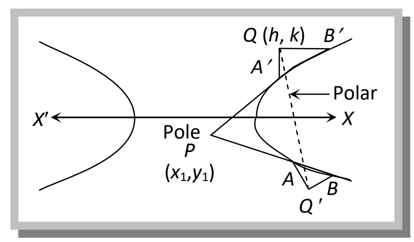
$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$



Note: Polar of the focus is the directrix.

Any tangent is the polar of its point of contact.

(1) **Pole of a given line :** The pole of a given line $lx + my + n = 0$ with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $(x_1, y_1) = \left(-\frac{a^2 l}{n}, \frac{b^2 m}{n}\right)$



(2) **Properties of pole and polar**

(i) If the polar of $P(x_1, y_1)$ passes through $Q(x_2, y_2)$, then the polar of $Q(x_2, y_2)$ goes through $P(x_1, y_1)$ and such points are said to be conjugate points.

(ii) If the pole of a line $lx + my + n = 0$ lies on the another line $l'x + m'y + n' = 0$ then the pole of the second line will lie on the first and such lines are said to be conjugate lines.

(iii) Pole of a given line is same as point of intersection of tangents as its extremities.

Important Tips

☞ *If the polars of* (x_1, y_1) *and* (x_2, y_2) *with respect to the hyperbola* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ *are at right angles,*

then $\frac{x_1 x_2}{y_1 y_2} + \frac{a^4}{b^4} = 0$