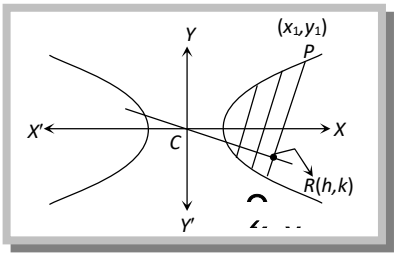
Diameter of the Hyperbola.

The locus of the middle points of a system of parallel chords of a hyperbola is called a diameter and the point where the diameter intersects the hyperbola is called the vertex of the diameter.



Let $y = mx + c$ a system of parallel chords to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ for different chords then the equation of diameter of the hyperbola is $y = \frac{b^2 x}{a^2 m}$, which is passing through (0, 0)

Conjugate diameter: Two diameters are said to be conjugate when each bisects all chords parallel to the others.

If $y = m_1 x$, $y = m_2 x$ be conjugate diameters, then $m_1 m_2 = \frac{b^2}{a^2}$.

Note: If a pair of diameters be conjugate with respect to a hyperbola, they are conjugate with respect to its conjugate hyperbola also.

In a pair of conjugate diameters of a hyperbola. Only one meets the curve in real points.

The condition for the lines $AX^2 + 2HXY + BY^2 = 0$ to be conjugate diameters of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $a^2 A = b^2 B$.

Important Tips

☞ *If* CD *is the conjugate diameter of* a *diameter CP of the hyperbola* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, *where P is* $(a\sec\phi, b\tan\phi)$ *then coordinates of D is* $(a\tan\phi, b\sec\phi)$, *where C is (0, 0).*