Asymptotes of a Hyperbola.

An asymptote to a curve is a straight line, at a finite distance from the origin, to which the tangent to a curve tends as the point of contact goes to infinity.

The equations of two asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are $y=\pm\frac{b}{a}x$ or $\frac{x}{a}\pm\frac{y}{b}=0$.

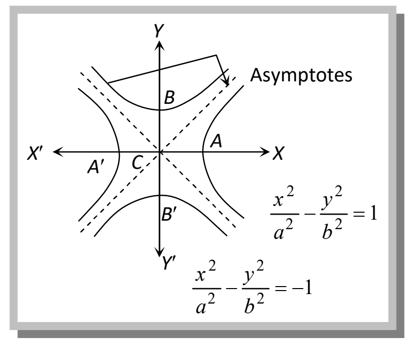
Note: The combined equation of the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$.

When $b=a$ *i.e.* the asymptotes of rectangular hyperbola $x^2-y^2=a^2$ are $y=\pm x$, which are at right angles.

A hyperbola and its conjugate hyperbola have the same asymptotes.

The equation of the pair of asymptotes differ the hyperbola and the conjugate hyperbola by the same constant only *i.e.* Hyperbola – Asymptotes = Asymptotes – Conjugated hyperbola or,

$$\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}-1\right)-\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)=\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)-\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}+1\right).$$



The asymptotes pass through the centre of the hyperbola.

The bisectors of the angles between the asymptotes are the coordinate axes.

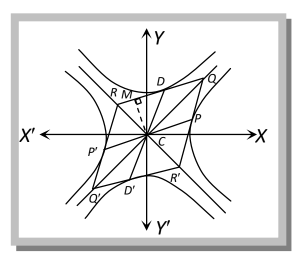
The angle between the asymptotes of the hyperbola $S=0$ *i.e.*, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\tan^{-1}\frac{b}{a}$ or $2\sec^{-1}e$.

Asymptotes are equally inclined to the axes of the hyperbola.

Important Tips

☞ *The parallelogram formed by the tangents at the extremities of conjugate diameters of a hyperbola has its vertices lying on the asymptotes and is of constant area.*

Area of parallelogram $QRQ'R'$ = *4(Area of parallelogram QDCP)* = $4ab$ = *Constant*



☞ *The product of length of perpendiculars drawn from any point on the hyperbola* $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ *to the asymptotes is* $\frac{a^2b^2}{a^2+b^2}$.